Stochastic Cooling

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http://cosmo.fnal.gov/organizationalchart/mcginnis/Talks/Talks.htm

FNAL Antiproton Source

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Why Stochastic Cooling

- Highest energies obtained in proton synchrotrons
- A single ring can contain two counter-rotating oppositely charged beams
- An intense antiproton source is needed for adequate luminosity
- Secondary beams of antiprotons are produced by targeting an intense proton beam
  - Intensities of secondary antiproton beams are 10^{-5} times smaller than proton production beams
  - Need to accumulate 10^5 antiproton pulses
  - Need to compress phase space of each antiproton pulse by a factor of 10^5.

Electron Cooling

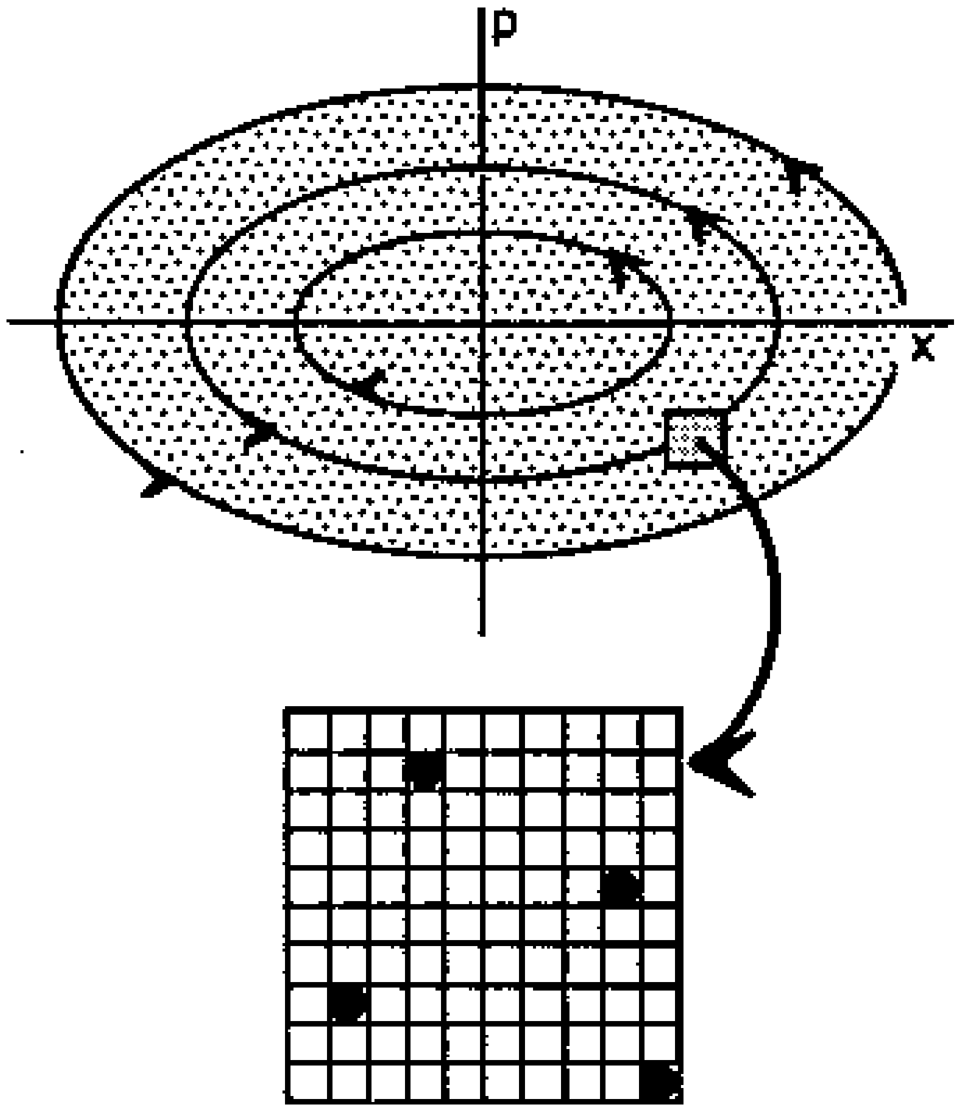
- Proposed by Budker in 1966 & demonstrated by Budker in 1976
- Effective with
  - Small size beams
  - High density beams
- “Cold” dense electron beam merged with “hot” beam
  - Both beams must have same average velocity
  - Cooling proportional to ratio of particle mass to electron mass
  - Analogous to placing a cold cup of coffee next to a hot cup of coffee

Stochastic Cooling

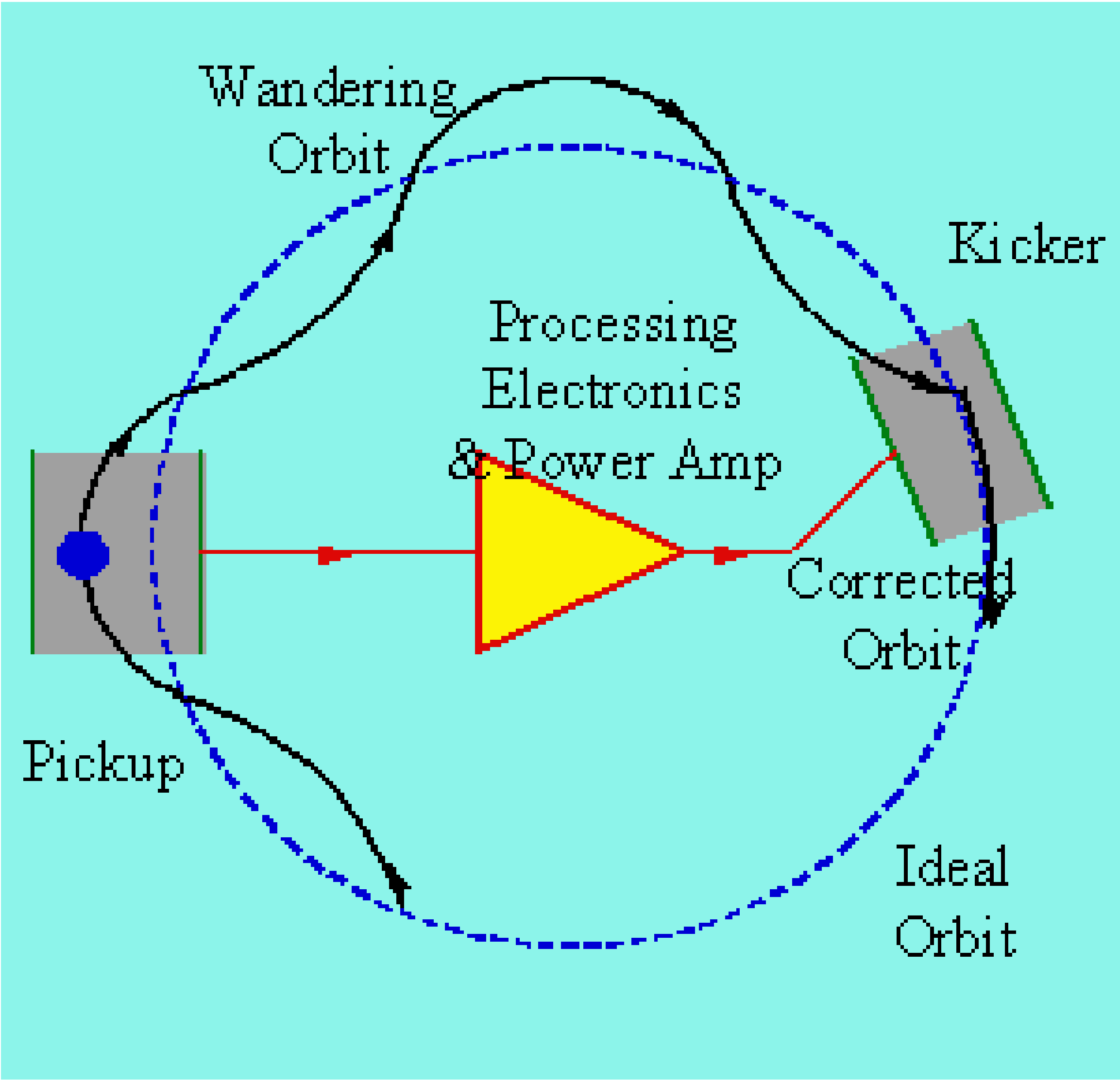
- Proposed by van der Meer in 1972. Demonstrated in the CERN ISR in 1975
- Effective with
  - ❑ Large size beams
  - ❑ Low density beams
- Each particle is placed on the "correct" orbit one by one using feedback electronics
  - ❑ Cooling times can be as short as seconds (FNAL Debuncher) and as long as tens of minutes (FNAL Accumulator)
  - ❑ Analogous to separating green paint into yellow and blue paint.

Stochastic Cooling Phase Space

- Stochastic cooling re-arranges phase space by placing particles into the “empty holes” in phase space.



Stochastic Cooling System Schematic



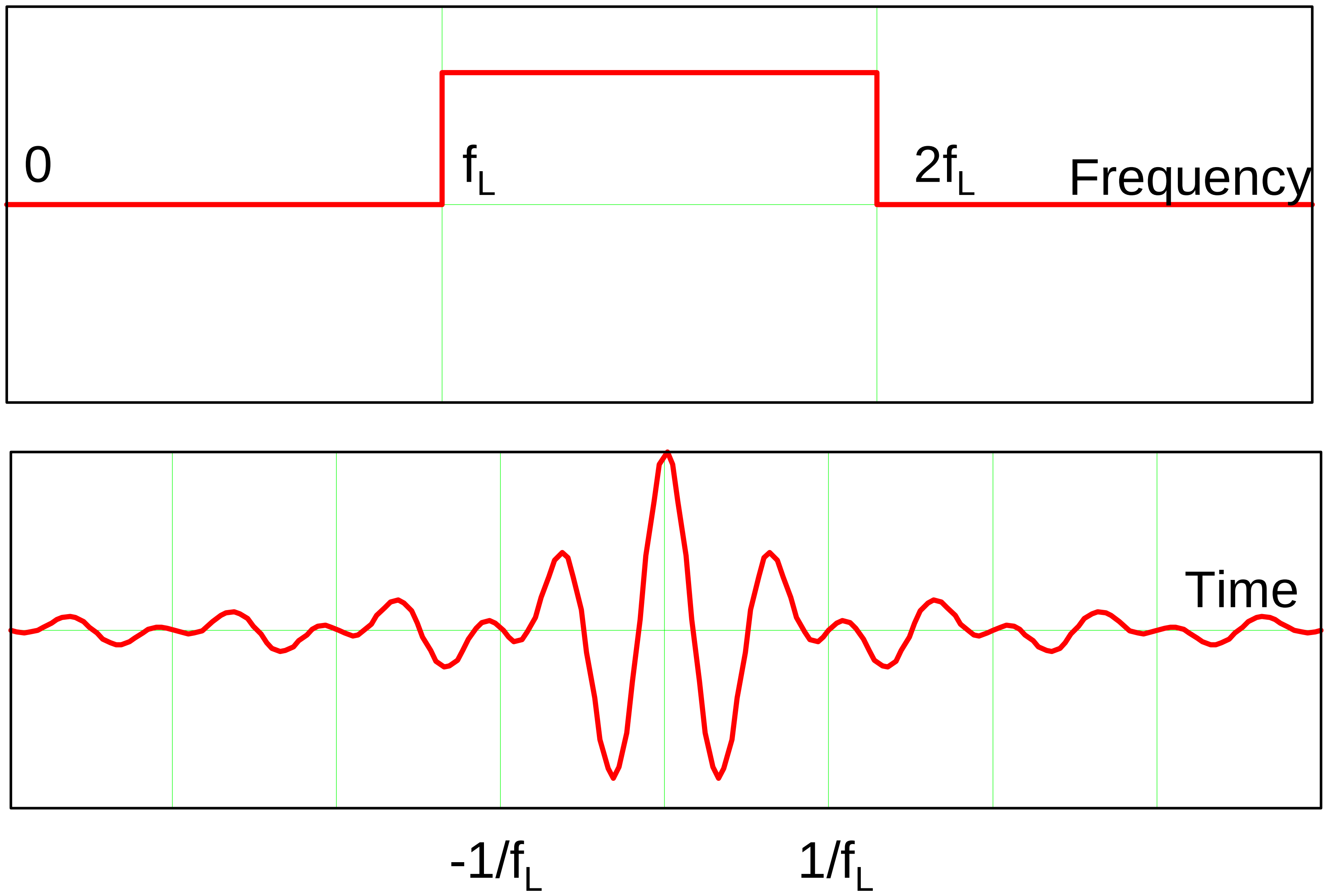
Stochastic Cooling uses Feedback

- A pickup electrode measures an error signal for a given particle
  - The error signal could be the particle's position or energy
  - The pickup signal can be extremely small - on the order of 2×10^{-12} Watt
- This signal is processed and amplified. The gain of a typical system is 150 dB (a factor of 10^{15} in power)
- The opposite of the error signal is applied to the particle at the kicker
  - The kicker signal can be as large as 2000 Watts
  - The kicker is usually similar in shape to the pickup and uses principle of Lorentz reciprocity. (The transmitting pattern is the same as the receiving pattern.)

System Bandwidth

- The resolution of a stochastic cooling system to resolve an individual particle is proportional to the bandwidth of the system.
- The size of the pickups is inversely proportional to the bandwidth of the pickups
- Bandwidth choice in the microwave regime of 1-10 GHz
  - Reasonable size pickups with respect to beam pipe aperture
  - Availability of Power Amplifiers
  - Bad Mixing (to be covered later…)
- Large bandwidths in the microwave regime are usually limited to an octave ($F_{upper} = 2 \times F_{lower}$)
  - Power amps, pickup design, etc..

Octave Bandwidth Response



Mixing

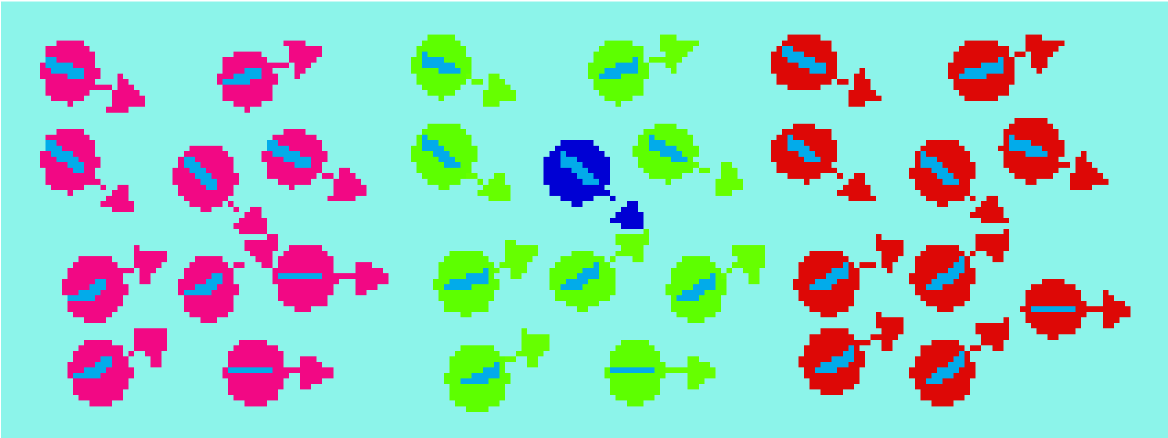
- The ability of the pickup to resolve a single particle is proportional to the bandwidth of the pickup
- To resolve a single particle in the FNAL Accumulator, the pickup bandwidth would have to be greater than 629,000 TeraHertz
- The maximum bandwidth of the cooling systems in the Accumulator is 4 GHz, so on average, there are 160×10^6 particles are underneath the pickup at any given time.

Mixing

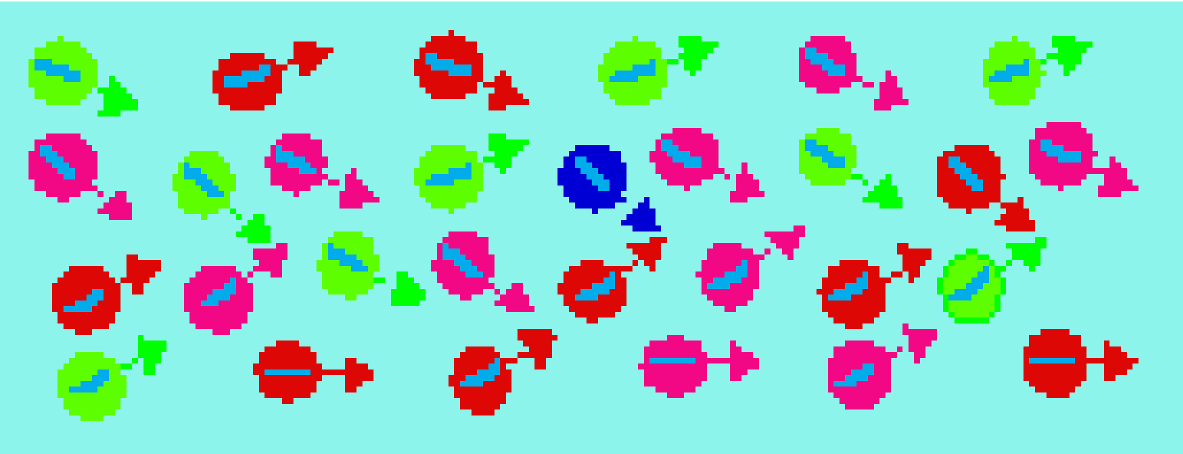
- These other particles are sources of noise for a given particle that needs to be cooled. But:
  - Since each particle has a slightly different energy than any other particle, every particle will take a slightly different time to travel around the accelerator
  - This causes the particles to continually mix up so that the noise contribution of the other particles averages to zero in the long run.
  - This effect is caused Mixing (Good Mixing). The Mixing factor is given as how many turns around the accelerator does it take for the beam to randomize.
    - FNAL Accumulator M=4 @ 6 GHz
    - FNAL Debuncher M=8 @ 6 GHz

Mixing

Before Mixing



After Mixing



Simple Cooling Theory

N_p = Number of particles in the accelerator

N_s = Number of particles underneath the pickup at any given time

T_r = Average revolution period of a particle

W = Bandwidth of the amplifier

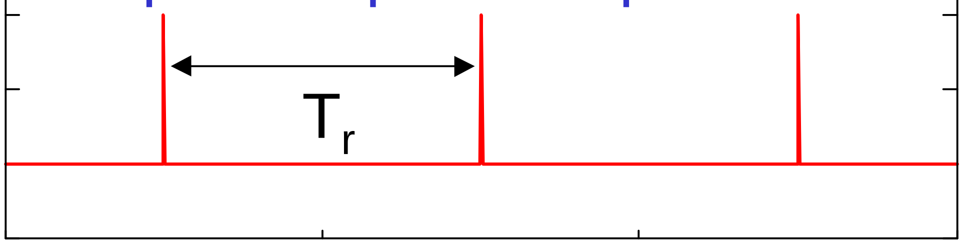
g_0 = Frequency gain of amplifier at 0 Hz

$x_i(m)$ = Transverse position w.r.t. the closed orbit of particle i on turn m

σ = r.m.s. beam size

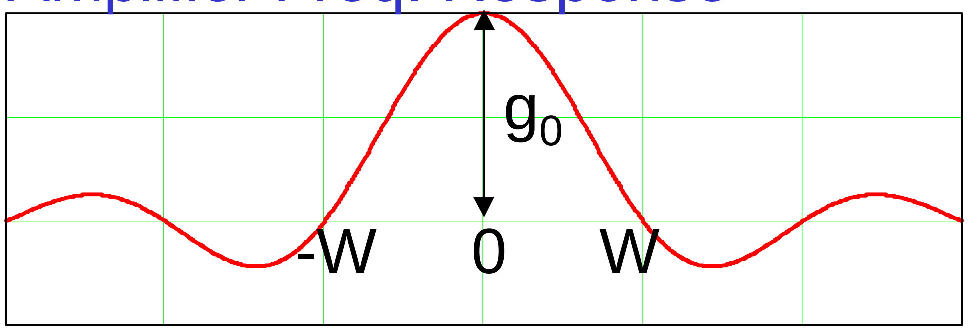
Simple Amplifier Response

Amplifier Input for 1 particle



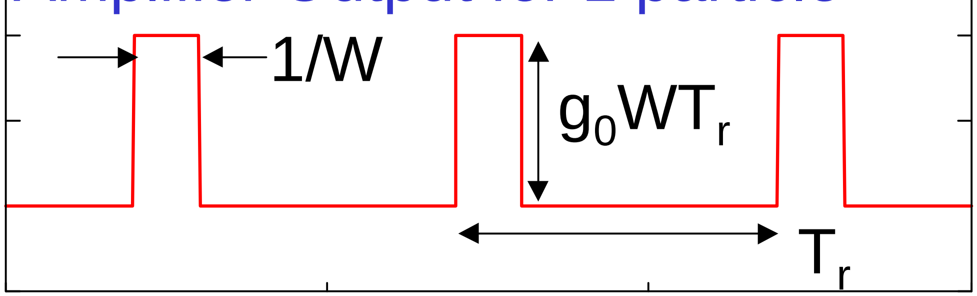
$$g(t) = \sum_{-\infty}^{\infty} g_n e^{jn\frac{2\pi}{T_r}t}$$

Amplifier Freq. Response



$$g_n = \frac{1}{T_r}\int_{-T_r/2}^{T_r/2} g(t)e^{-jn\frac{2\pi}{T_r}t}\,dt$$

Amplifier Output for 1 particle



$$g_n = g_0\frac{\mathrm{Sin}\left(\frac{n\omega_r}{2W}\right)}{n\omega_r/2W}$$

Simple Cooling Theory

Correction at kicker for particle i on turn m+1

$$\boxed{x_i(m+1) = x_i(m) - g_0 W T_r \sum_{j=i-N_s/2}^{i+N_s/2} x_j(m)}$$

Number of particles underneath pickup/kicker

$$\boxed{N_s = N_p \frac{1/W}{T_r} = \frac{N_p}{W T_r}}$$

Simple Cooling Theory

Square both sides of equation

$$\left(x_i(m+1)\right)^2-\left(x_i(m)\right)^2=-2g_0WT_r x_i(m)\sum_{j=i-N_s/2}^{i+N_s/2}x_j(m)$$
$$+g_0{}^2(WT_r)^2\sum_{j=i-N_s/2}^{i+N_s/2}x_j(m)\sum_{k=i-N_s/2}^{i+N_s/2}x_k(m)$$

Sum over all particles

$$\frac{1}{N_p}\sum_{i=1}^{N_p}\left(x_i(m+1)\right)^2-\left(x_i(m)\right)^2=-2g_0WT_r\frac{1}{N_p}\sum_{i=1}^{N_p}x_i(m)\sum_{j=i-N_s/2}^{i+N_s/2}x_j(m)$$
$$+g_0{}^2(WT_r)^2\frac{1}{N_p}\sum_{i=1}^{N_p}\sum_{j=i-N_s/2}^{i+N_s/2}x_j(m)\sum_{k=i-N_s/2}^{i+N_s/2}x_k(m)$$

Simple Cooling Theory

Averaging <> over many turns and assuming mixing in one turn

$$\frac{1}{N_p}\left\langle \sum_{i=1}^{N_p} x_i(m) \sum_{j=i-N_s/2}^{i+N_s/2} x_j(m) \right\rangle = \frac{1}{N_p}\sum_{i=1}^{N_p}(x_i)^2 = \sigma^2$$

$$\frac{1}{N_p}\left\langle \sum_{i=1}^{N_p}(x_i(m+1))^2 - (x_i(m))^2 \right\rangle = \Delta\sigma^2$$

$$\left\langle \sum_{j=i-N_s/2}^{i+N_s/2} x_j(m) \sum_{k=i-N_s/2}^{i+N_s/2} x_k(m) \right\rangle = \sum_{j=i-N_s/2}^{i+N_s/2}(x_j)^2 = N_s\sigma^2$$

Simple Cooling Theory

The average change in beam size over 1 turn

$$\boxed{\Delta\sigma^2 = \left(-2g_0 + g_0{}^2 N_p\right) W T_r \sigma^2}$$

The average value of the beam size will change over time by:

$$\boxed{\frac{d\sigma^2}{dt} = W\left(-2g_0 + g_0{}^2 N_p\right)\sigma^2}$$

which has the following solution

$$\boxed{\sigma^2 = \sigma_0{}^2 e^{-t/\tau}}$$

Simple Cooling Theory

The cooling rate is:

$$\boxed{\frac{1}{\tau} = W\left(2g_0 - g_0{}^2 N_p\right)}$$

The fastest cooling rate occurs when:

$$\boxed{g_0\Big|_{opt} = \frac{1}{N_p}}$$

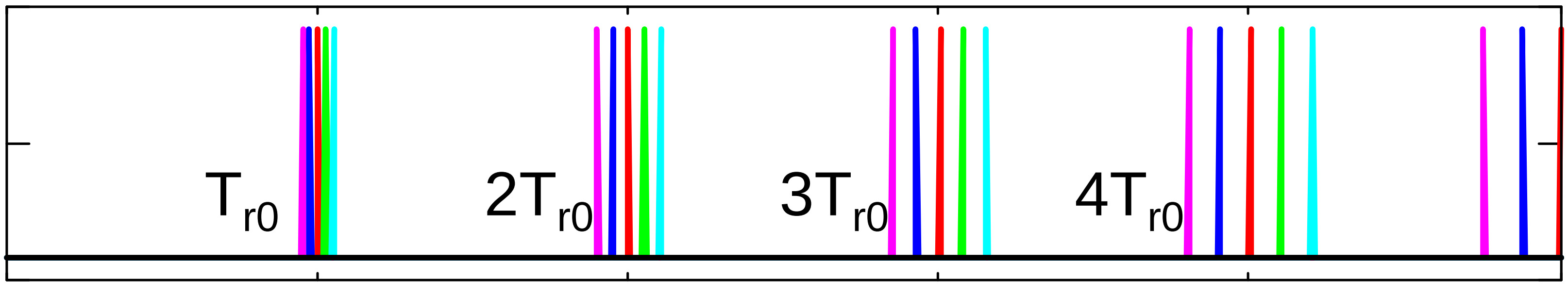
The fastest cooling rate is:

$$\boxed{\frac{1}{\tau}\Big|_{opt} = \frac{W}{N_p}}$$

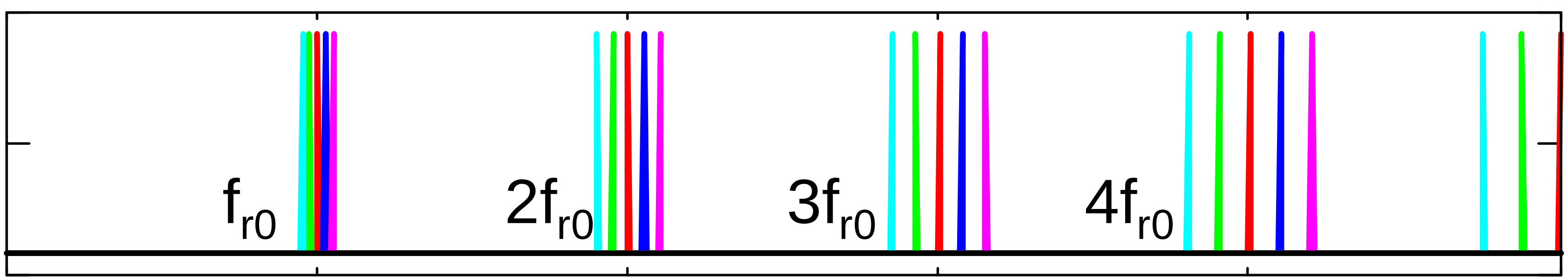
Schottky Signals

$$-\eta\frac{\Delta pc}{pc} = -\frac{\Delta T_r}{T_r} = \frac{\Delta f_r}{f_r} = \frac{\Delta h f_r}{h f_r}$$

$$i_k(t) = q\sum_{n=-\infty}^{\infty}\delta\left(t - nT_{r_k} - \tau_k\right)$$



$$i_k(t) = \sum_{n=-\infty}^{\infty} i_k^{(n)}\left(n\omega_{r_k} t\right) = \frac{q}{T_{r_k}}\sum_{n=-\infty}^{\infty} e^{jn\frac{2\pi}{T_{r_k}}(t-\tau_k)}$$



Schottky Signals

- Due to momentum spread in the beam, each particle revolves around the accelerator with a slightly different revolution frequency
- This periodic motion will present a periodic spectrum in frequency space. The periods of the spectrums are called bands.
- At a given band, the instantaneous current due to two different particles will sometimes add and sometimes subtract from one another because of the small difference in revolution frequencies.
- On average, the interaction between the two particles will cancel out and the POWER at a band (not the current!!!) will be the sum of the power from each particle

Schottky Signals

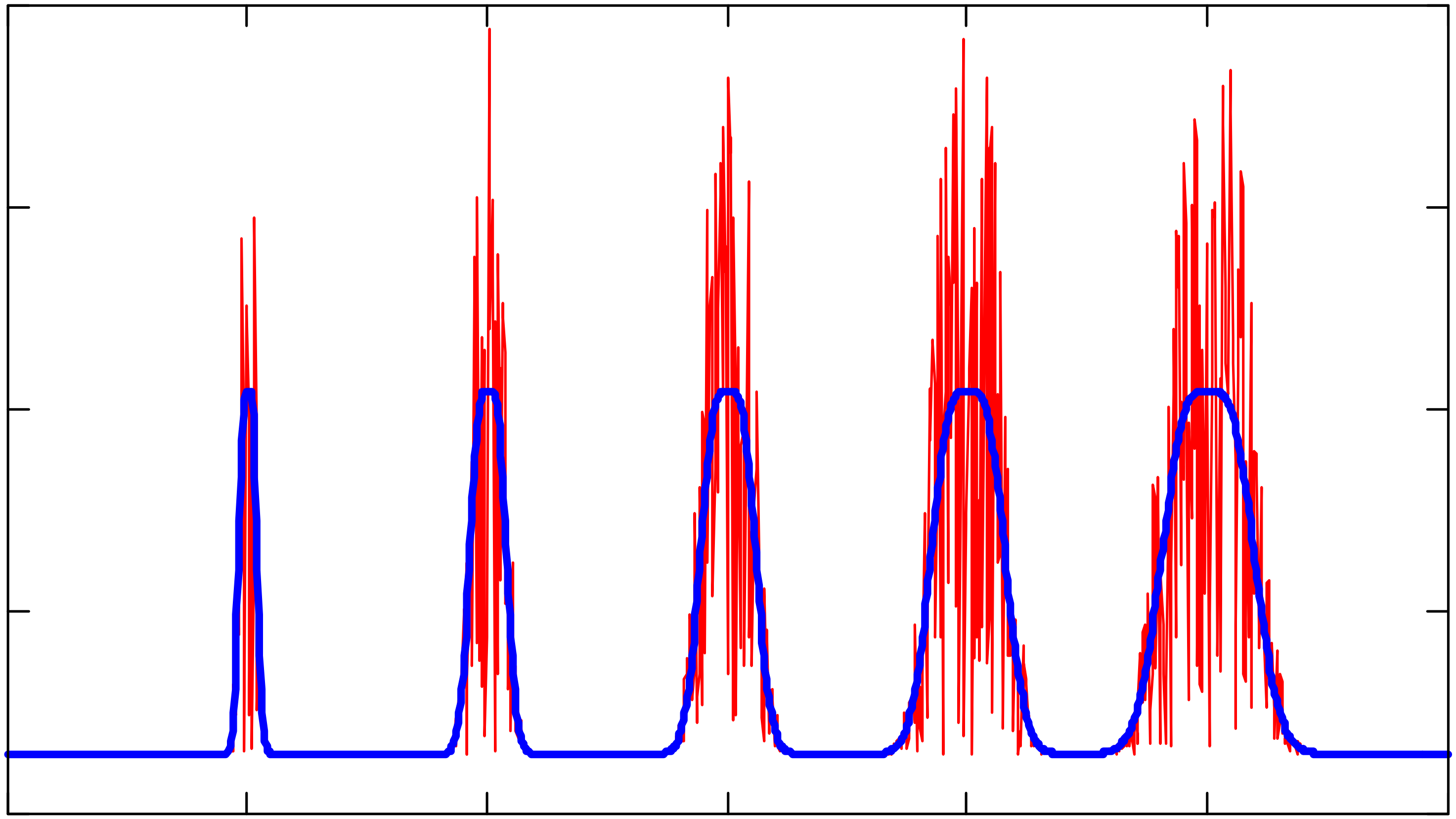
The power at band **n** due to particle **k**

$$P_k^{(n)} = Z_{pu}\left\langle i_k^{(n)}\left(n\omega_{r_k} t\right)\right\rangle = Z_{pu}\left(\frac{q}{T_{r_k}}\right)^2$$

The total power at band **n**

$$P^{(n)} = \sum_{k=1}^{N_p} P_k^{(n)} = N_p Z_{pu}\left(\frac{q}{T_{r_k}}\right)^2$$

Schottky Spectrum

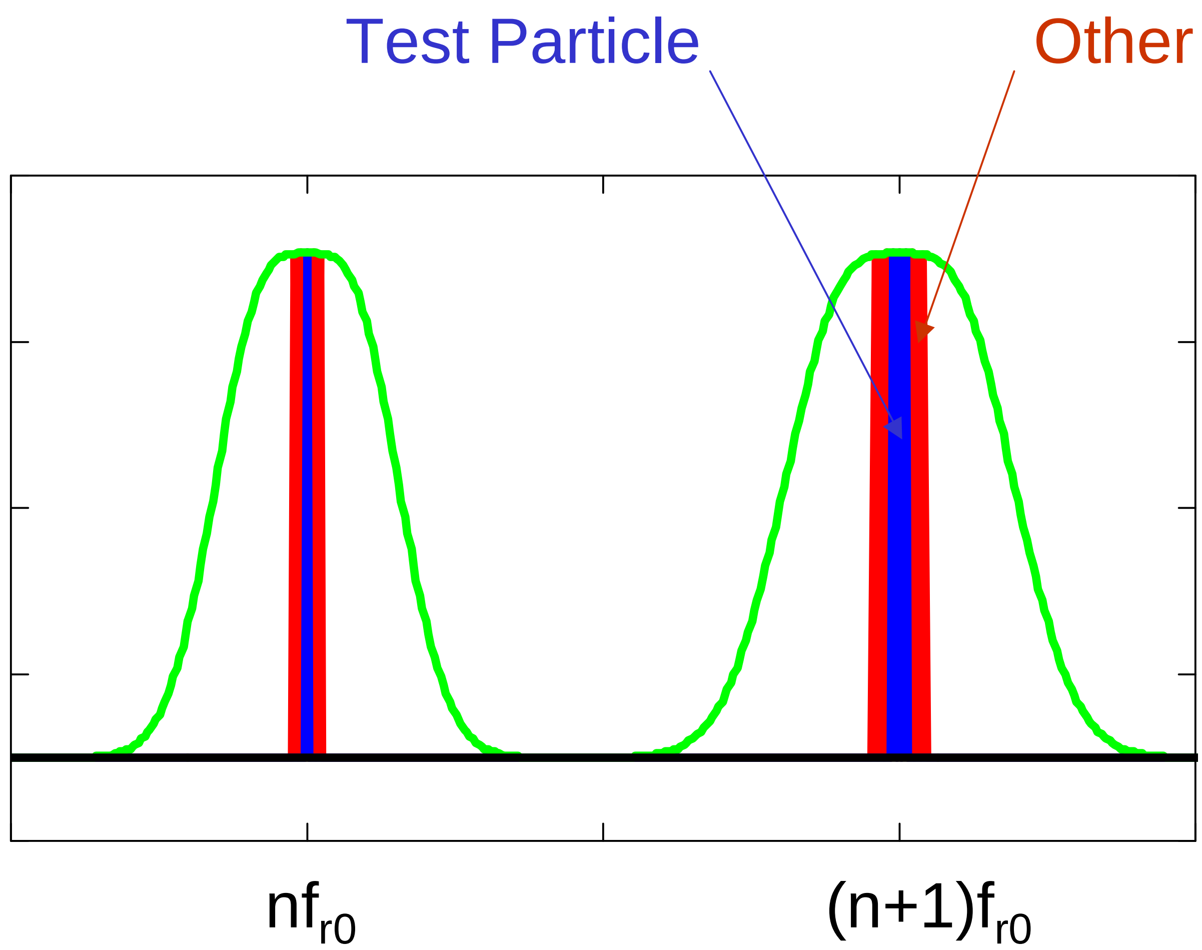


Short term spectrum

Long term spectrum

Mixing in the Frequency Domain

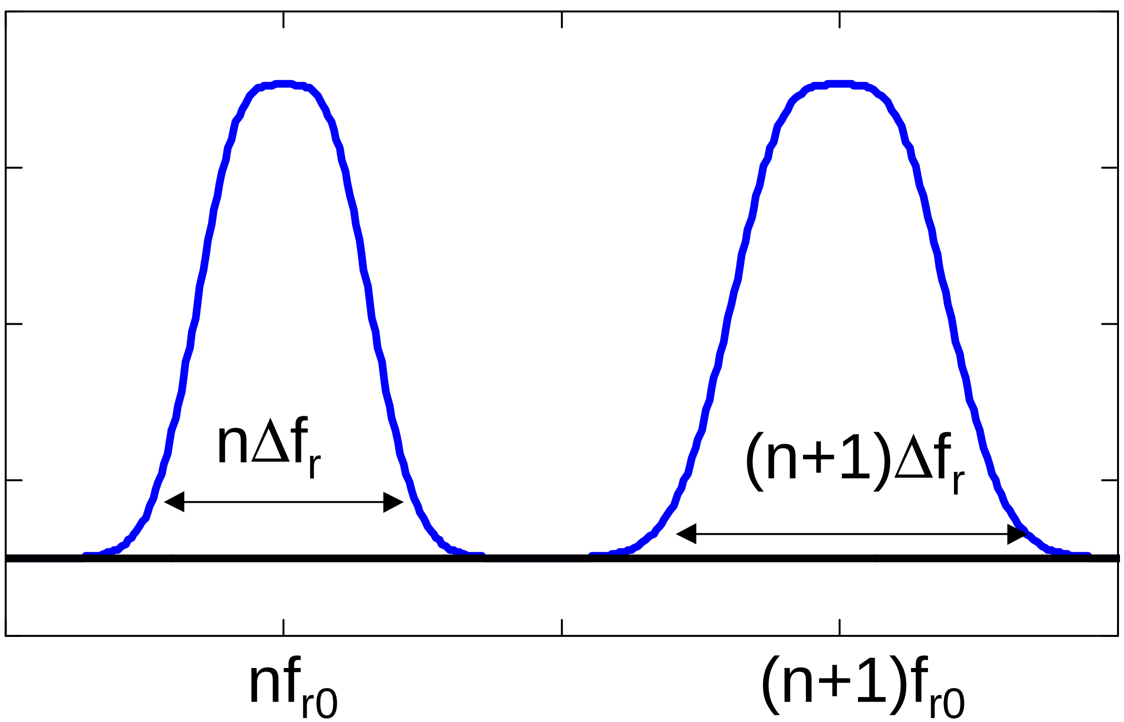
- At a given band, particles with revolution frequencies close to the particle we want to cool produce "noise" that heats the particle



Mixing in the Frequency Domain

- The mixing factor at a given harmonic **n** (band) is given as:

$$\boxed{M\left(nf_{r0}\right)=\frac{f_r}{n\Delta f_r}}$$



Cooling Rate with Mixing factor

The cooling rate is:

$$\boxed{\frac{1}{\tau} = W\left(2g_0 - g_0{}^2 N_p (M+U)\right)}$$

Where M is the average mixing factor of the system bandwidth and U is the average (thermal) Noise to Signal.

The fastest cooling rate occurs when:

$$\boxed{g_0\big|_{opt} = \frac{1}{N_p (M+U)}}$$

The fastest cooling rate is:

$$\boxed{\frac{1}{\tau}\bigg|_{opt} = \frac{W}{N_p (M+U)}}$$

Cooling Rate Examples

FNAL Accumulator Core
Transverse systems

$N_p = 1\text{x}10^{12}$

$W = 3\text{x}10^9$ Hz

$\Delta p/p = .15\%$

$\eta = 0.012$

$f_r = 628{,}888$ Hz

M(6GHz) = 5.8

$\tau = 32$ min

FNAL Debuncher
Transverse systems

$N_p = 85\text{x}10^6$

$W = 3\text{x}10^9$ Hz

$\Delta p/p = .5\%$

$\eta = 0.006$

$f_r = 590035$ Hz

M(6GHz) = 3.3

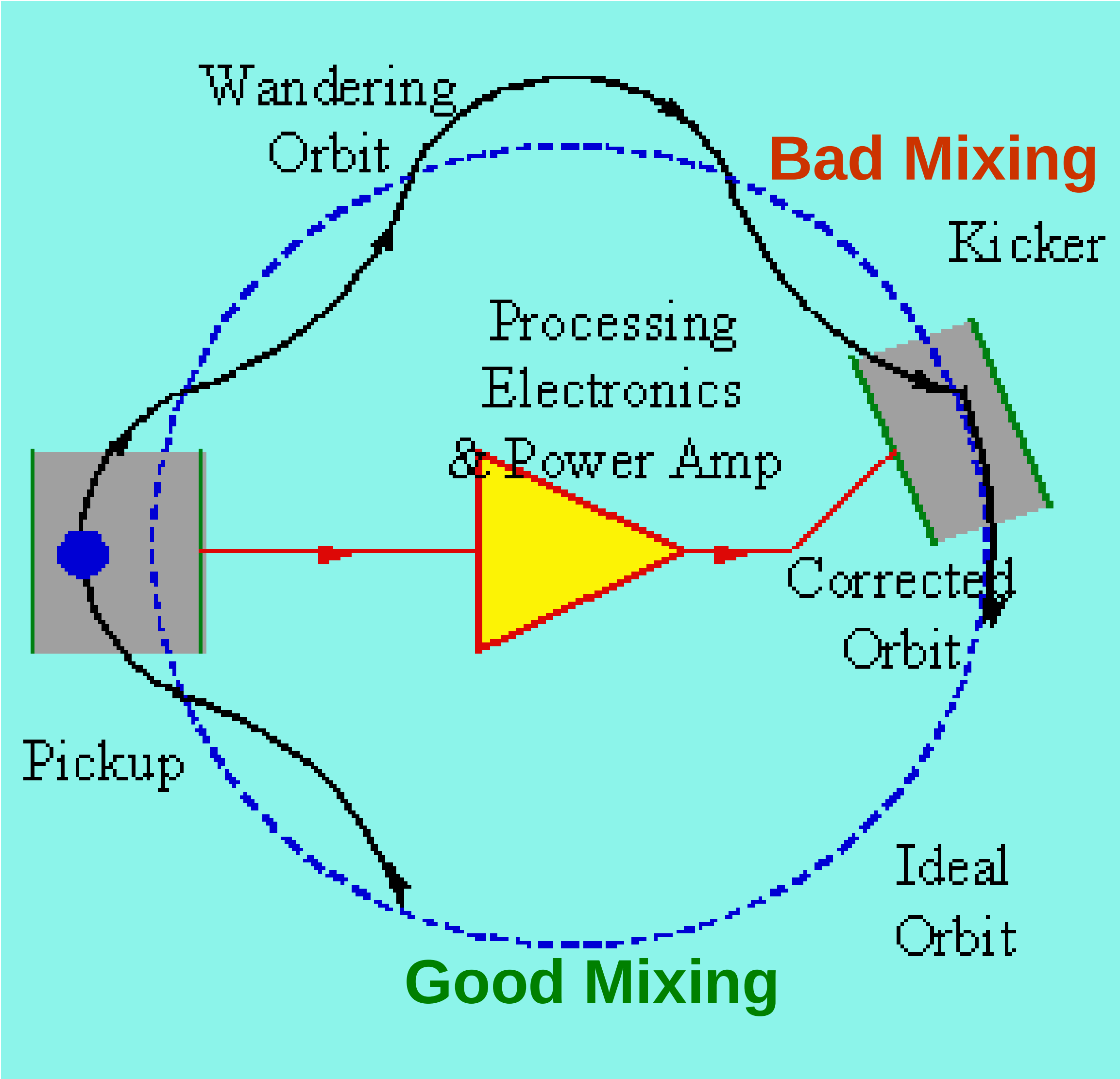
$\tau = .1$ sec !!!

τ(reality) = 1.5 sec

Bad Mixing

- We want the particles to randomize completely when going from KICKER to PICKUP
- Because of dispersion, two particles with different momenta will take different times to travel from PICKUP to KICKER
- The cable that connects pickup to kicker has the right delay for only one of these particles.
- The other particle will arrive out of phase with its cooling signal when it arrives at the kicker.
- This phase error gets worse at high frequencies

Bad Mixing



Wandering
Orbit
Bad Mixing
Kicker
Processing
Electronics
& Power Amp
Corrected
Orbit
Pickup
Ideal
Orbit
Good Mixing

Bad Mixing

The difference in transit time between pickup to kicker

$$\boxed{\Delta\tau = xT_r\eta\frac{\Delta p}{p}}$$

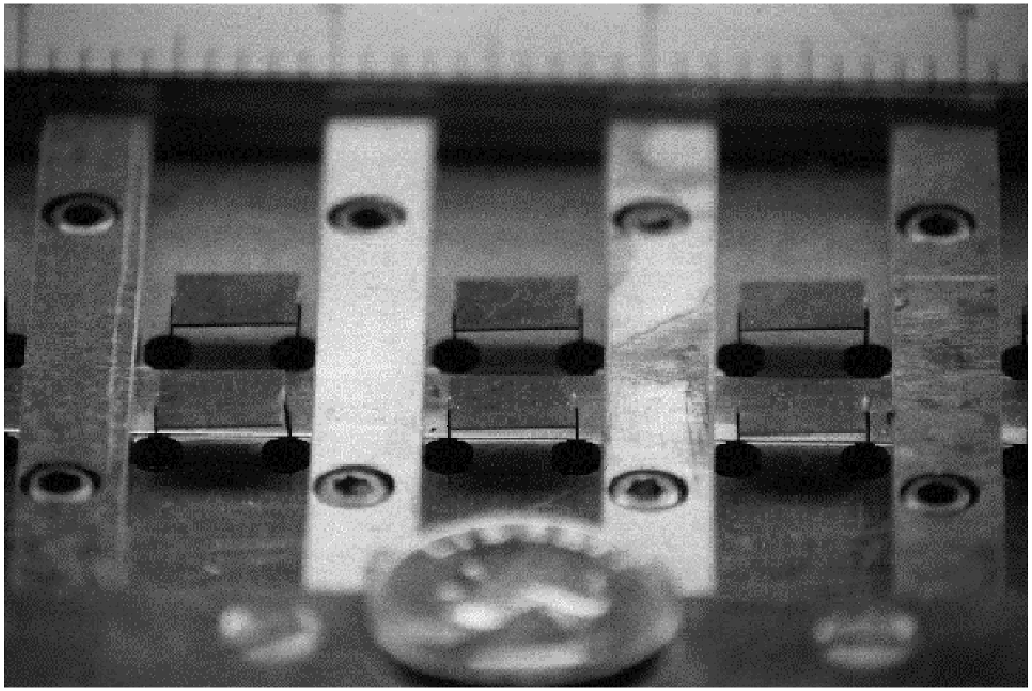
where **x** is the fraction of the circumference that the particles must travel from pickup to kicker:

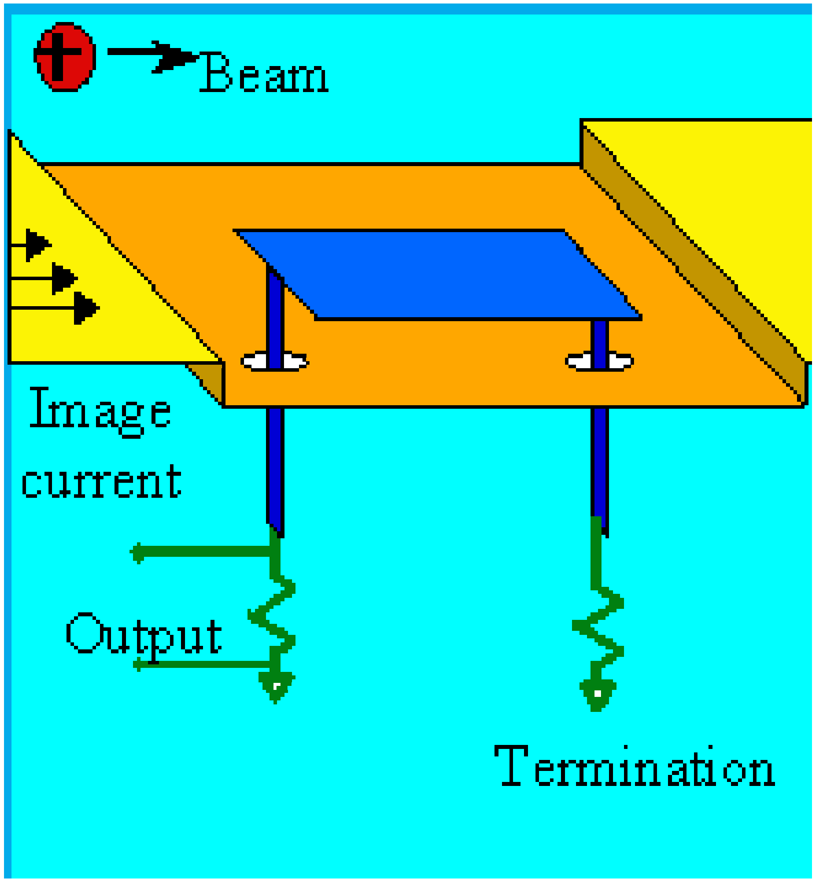
$$\boxed{\Delta\theta_{max} = 2\pi f_{max}\Delta\tau \le \frac{\pi}{2}}$$

Maximum frequency of the cooling system

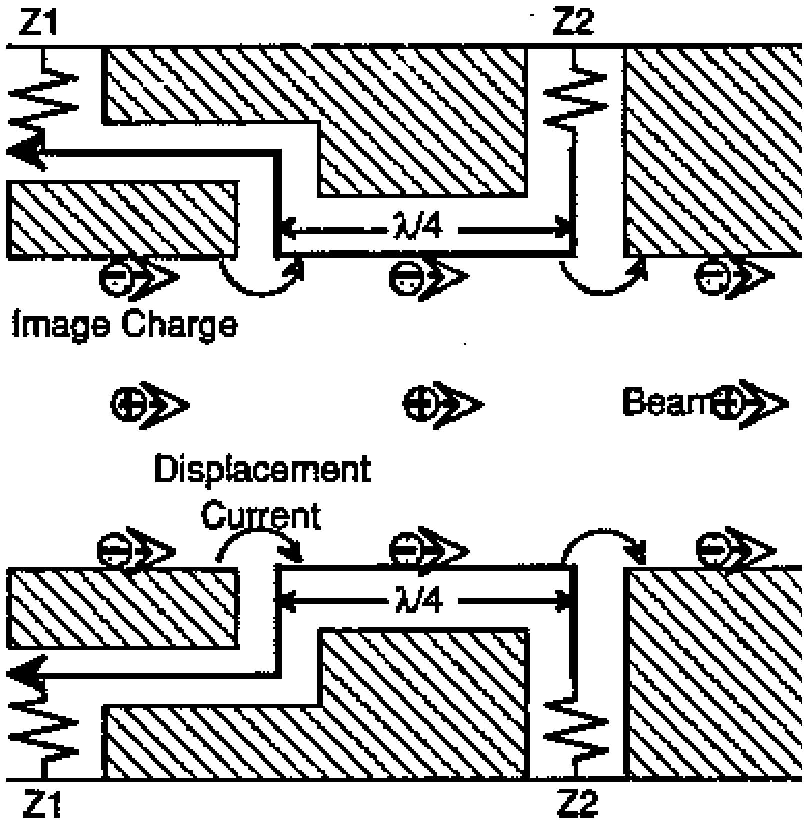
$$\boxed{f_{max} \le \frac{1}{4xT_r\eta\frac{\Delta p}{p}}}$$

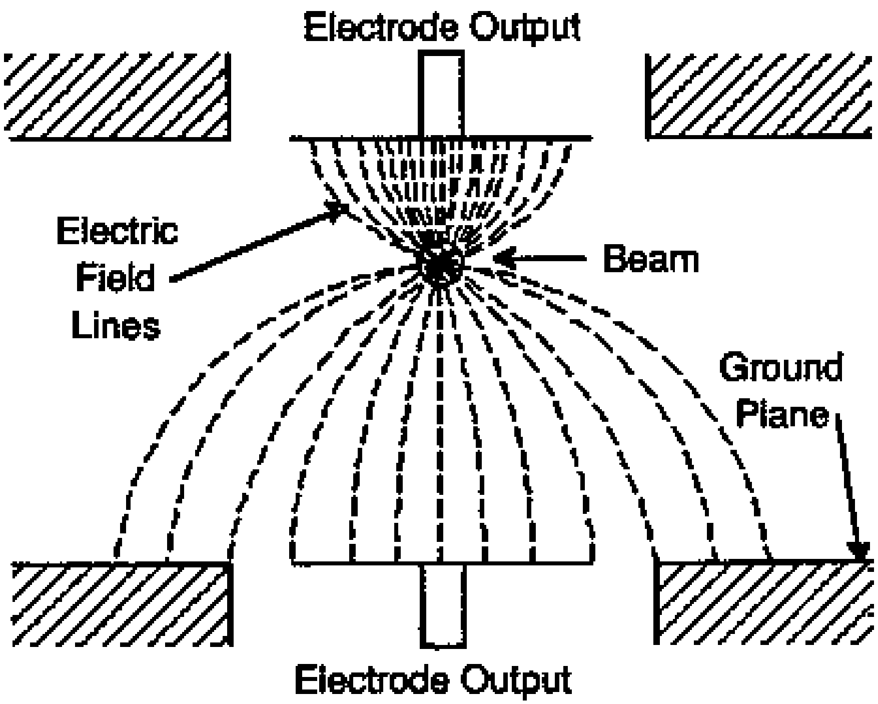
3D Stripline Pickups



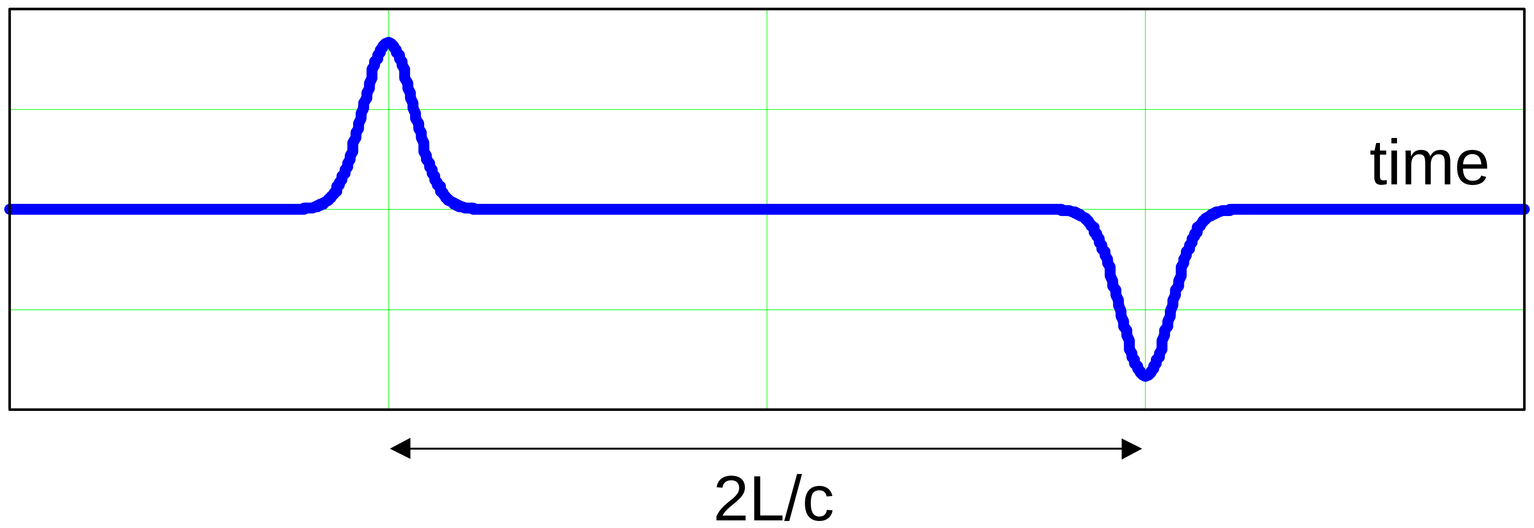


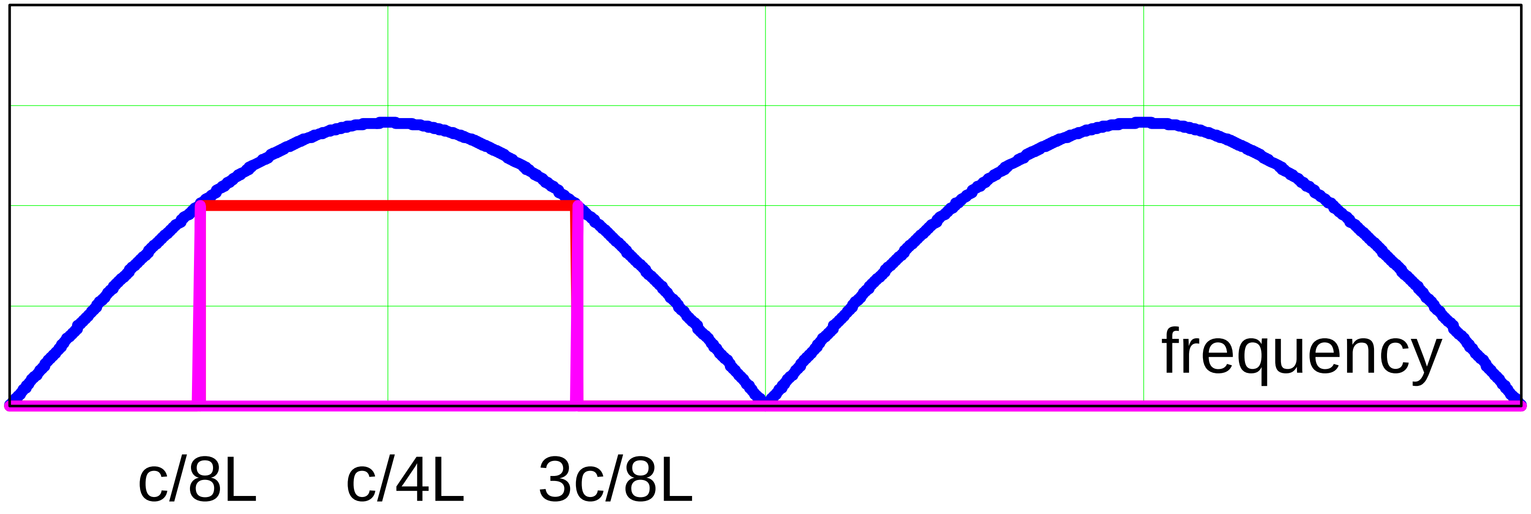
3D Stripline Pickups



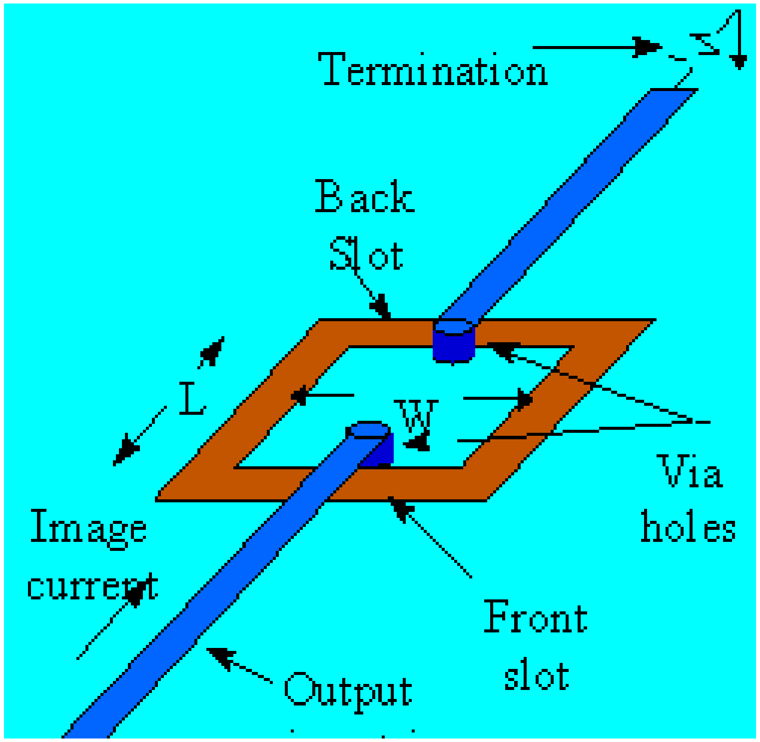


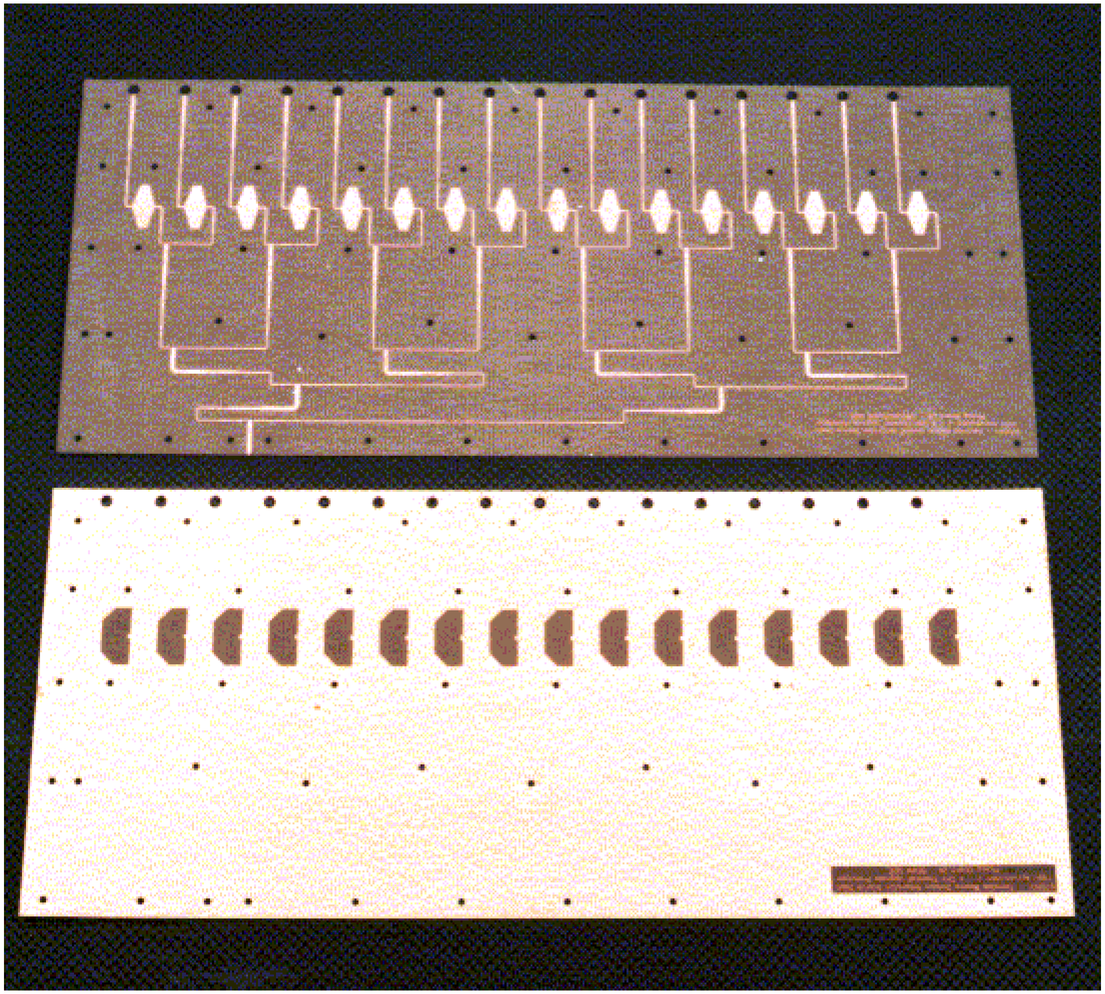
3D Stripline Pickups





Planar Loops



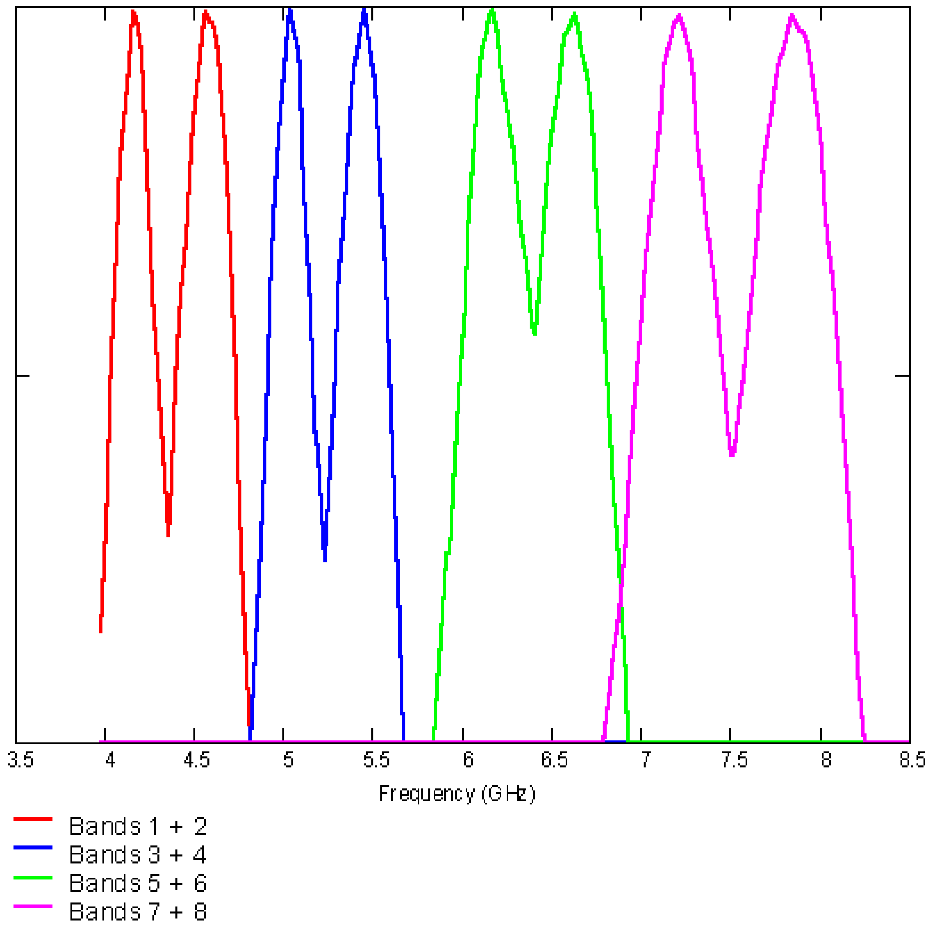


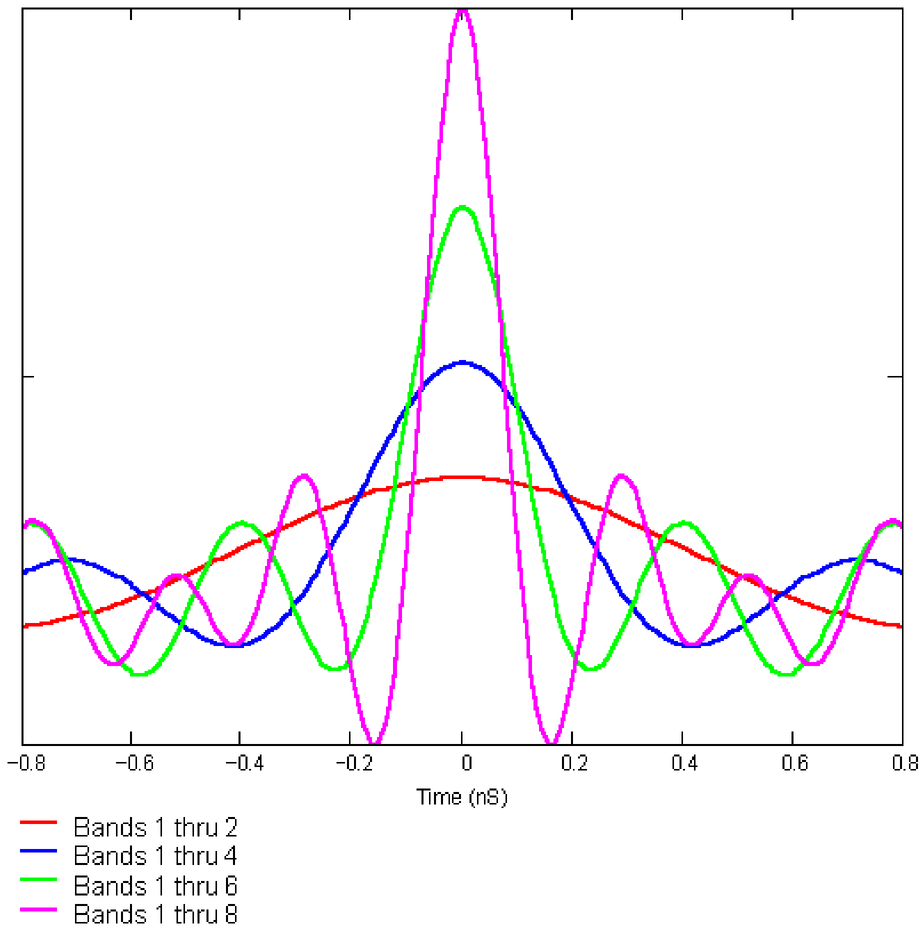
Combiner Boards

Multi-Band Band Cooling Systems

- At high frequencies, building wideband arrays becomes difficult because the beam pipe can support many microwave modes.
- The beam pipe over-moding problem can be overcome by breaking the cooling band into many narrow-band channels centered at different frequencies
- Sensitive narrow band pickup and kicker arrays can be constructed from slow wave structures
- The net result of many narrow band systems centered at adjacent frequencies is a high resolution, wideband system

Multi-Band Cooling Systems

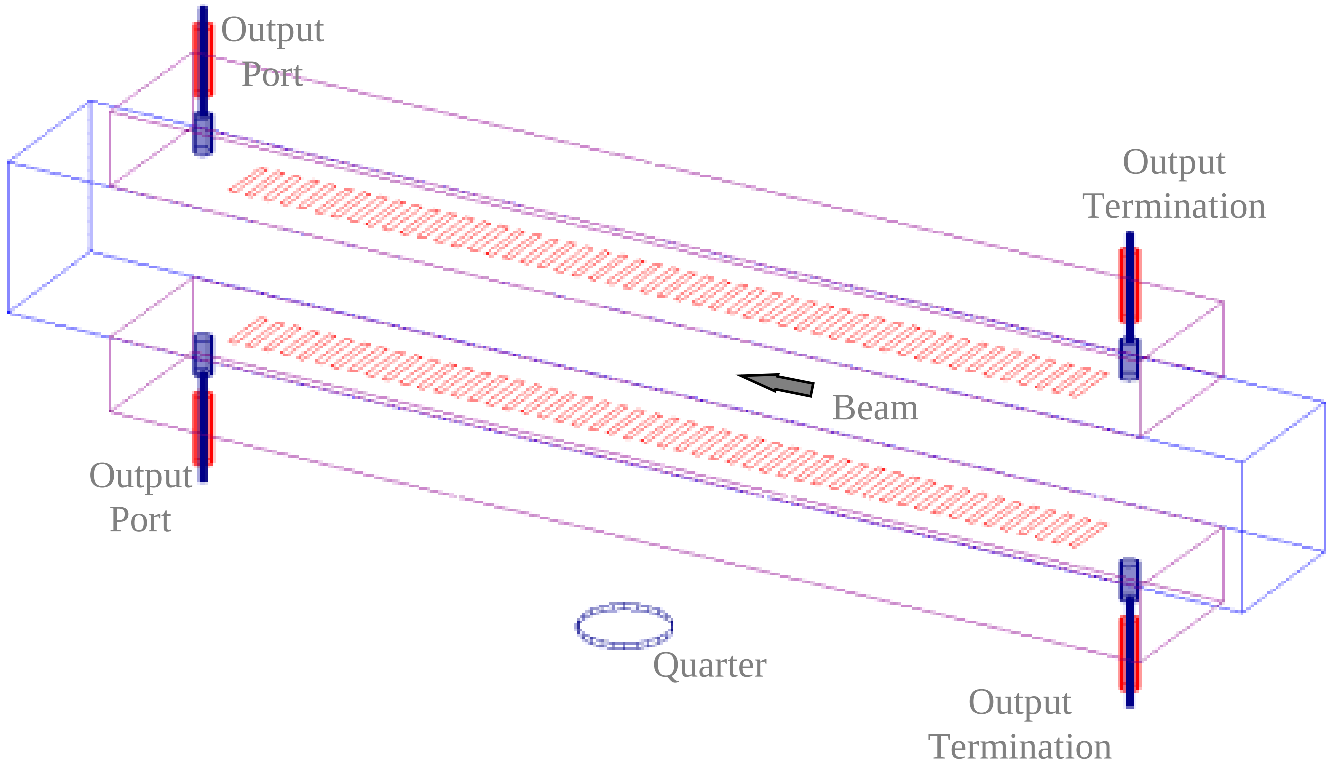




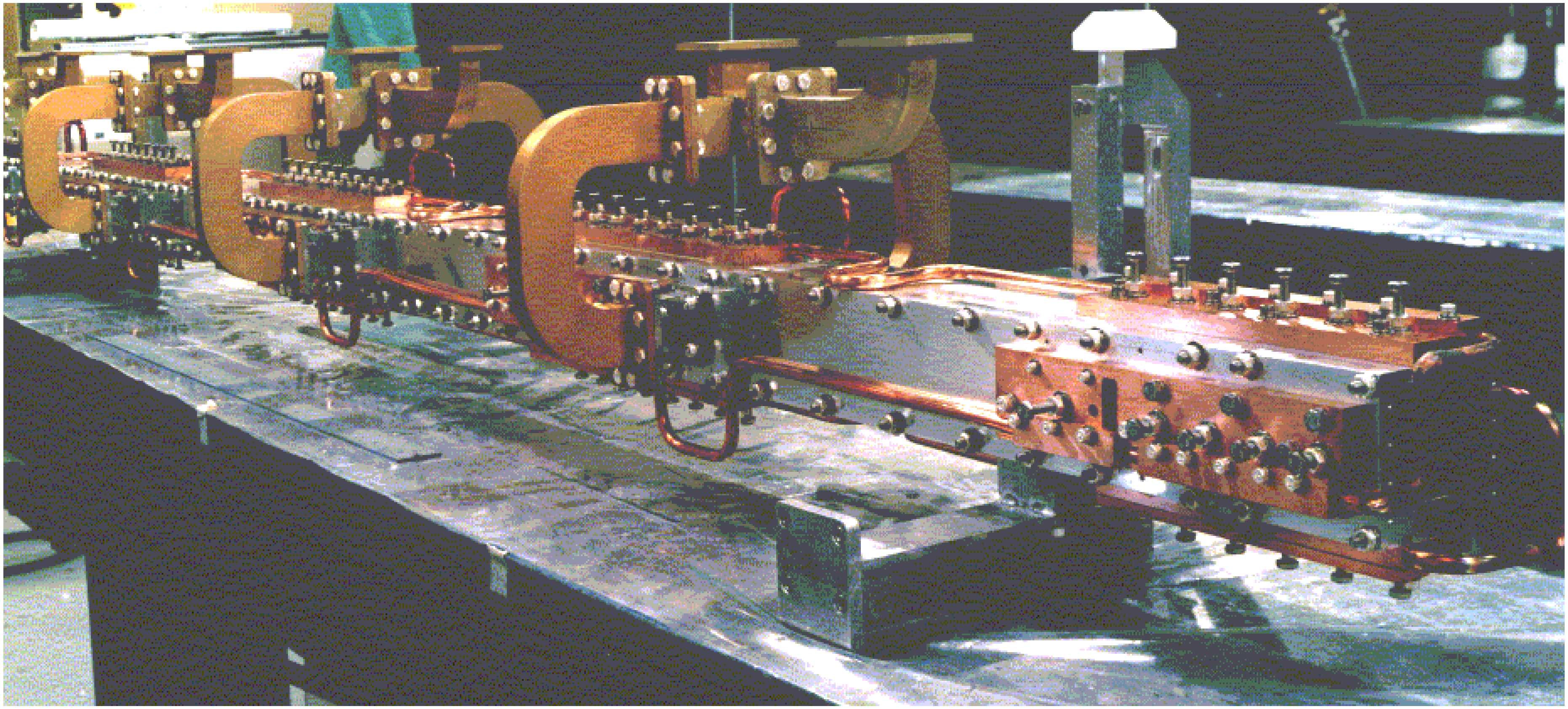
Slow Wave Pickups

- Slots carved in a waveguide wall will slow down the <u>phase</u> velocity of a wave in the waveguide.
- When the reduced phase velocity of the waveguide matches the beam velocity, the coupling of the slots will add constructively.
  - The gain of the array is proportional to the number of slots.
  - The bandwidth of the array is inversely proportional to the number of slots.

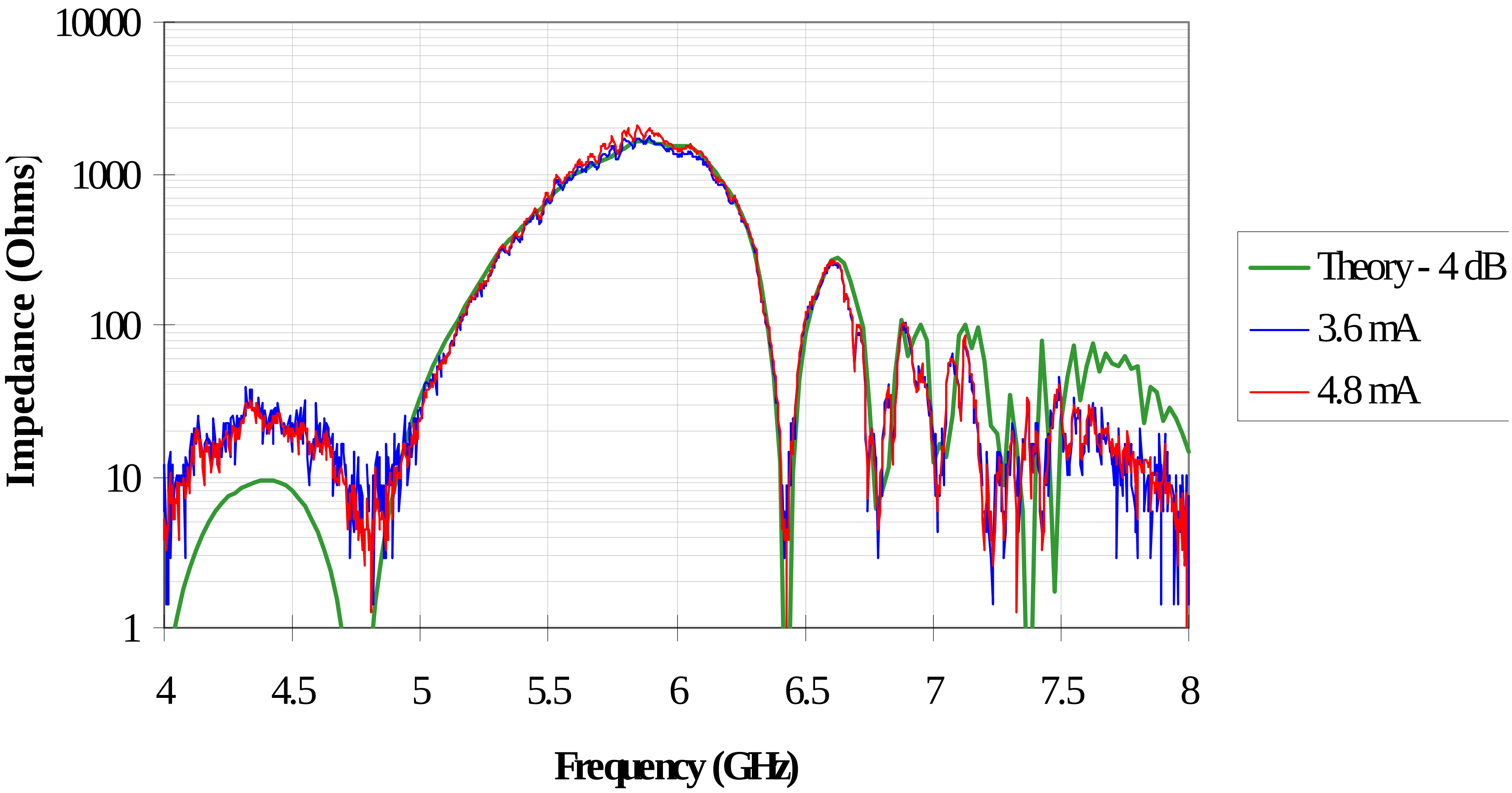
Slow Wave Array



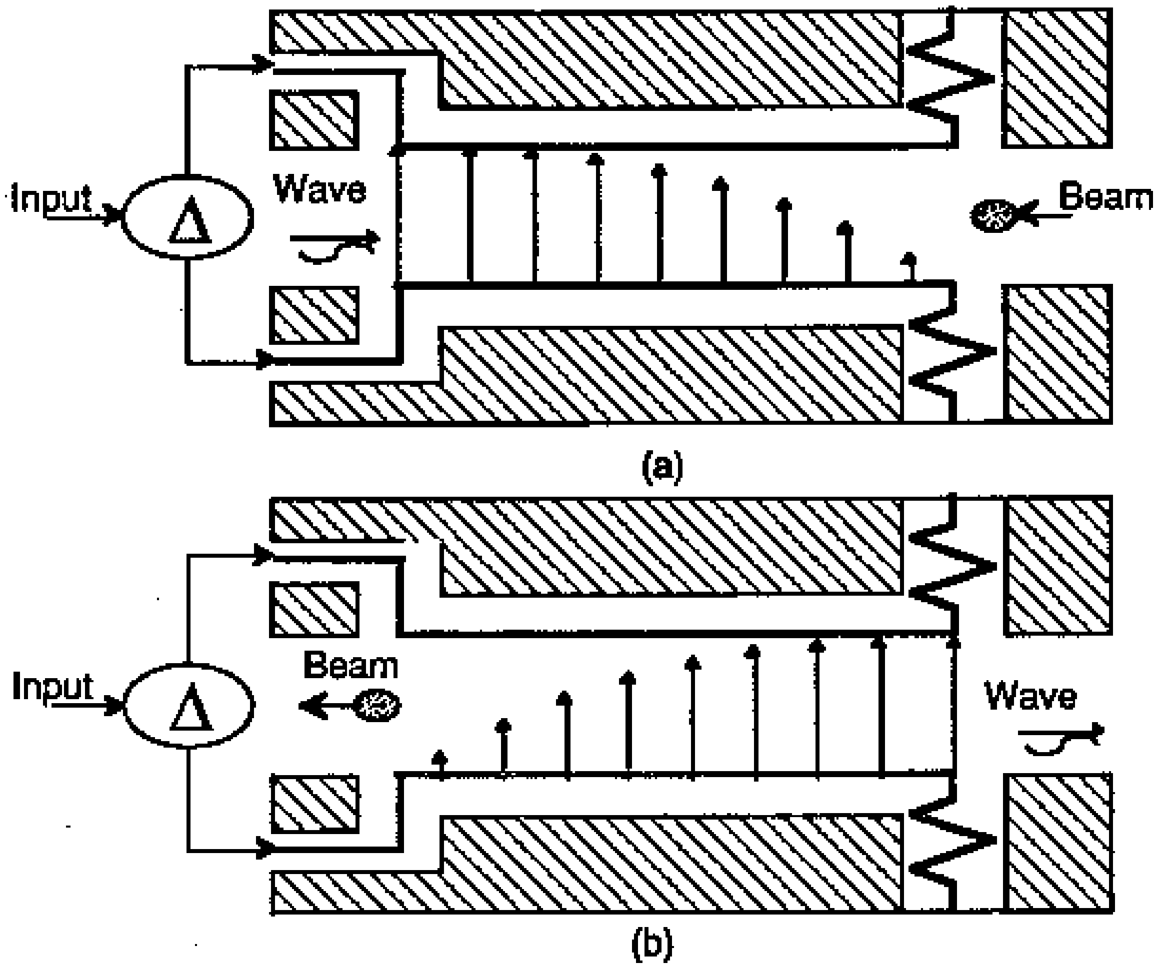
Slow Wave Array



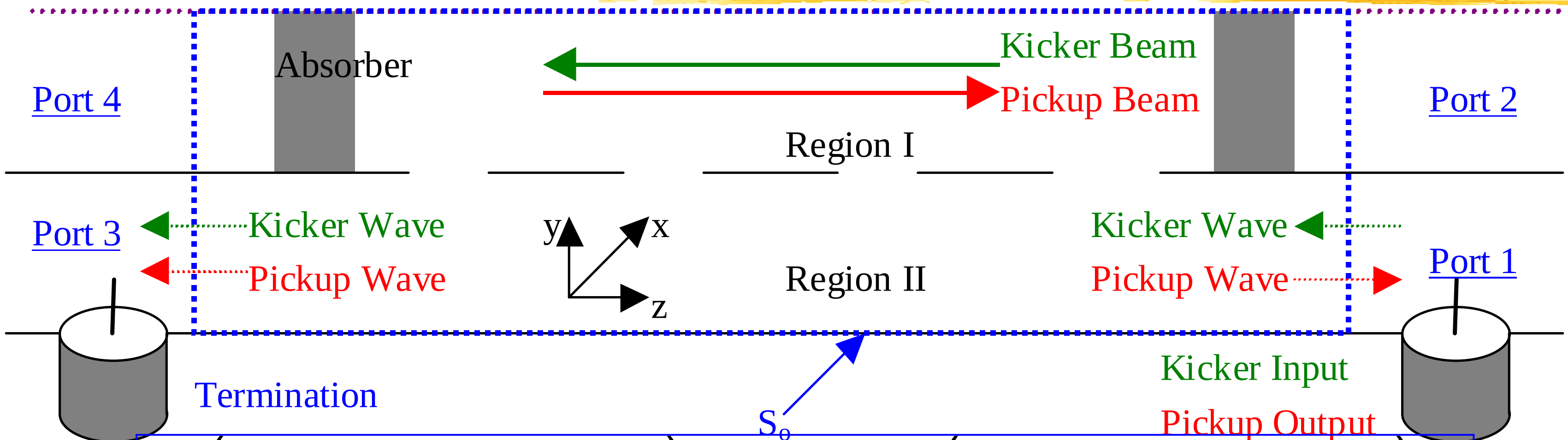
Slow Wave Array Response



3D Stripline Kicker



Reciprocity



$$\oiint_{S_o} \left(\vec{E}^p \times \vec{H}^k - \vec{E}^k \times \vec{H}^p \right) \bullet \hat{n} dS = \iiint_V \left(\vec{H}^k \bullet \vec{M}^p - \vec{E}^k \bullet \vec{J}^p \right) dv$$

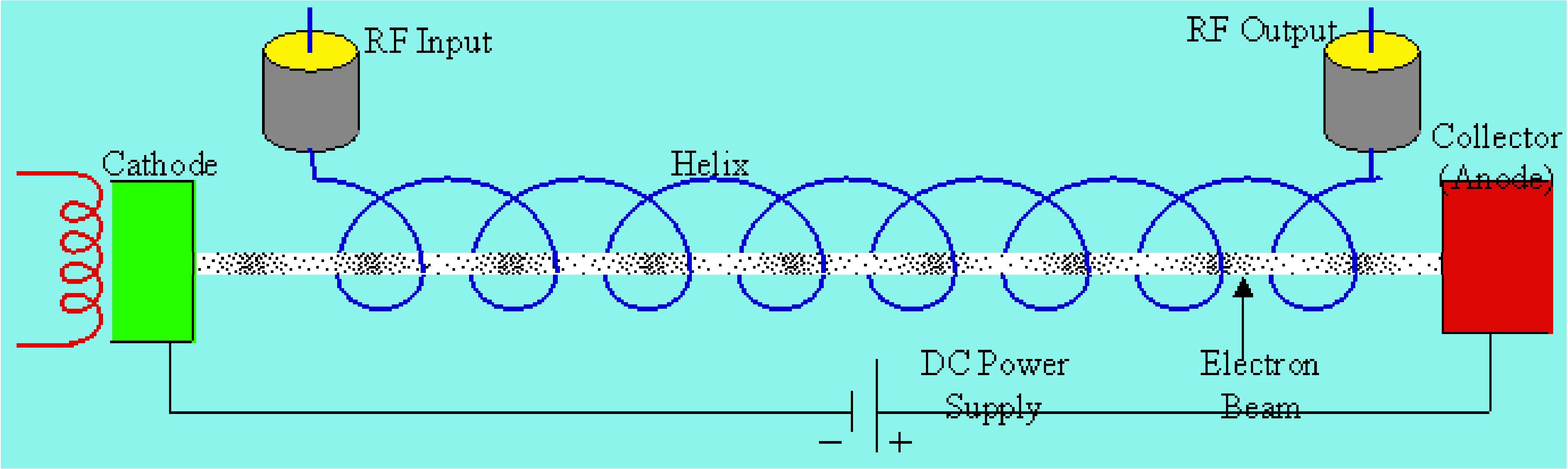
$$\frac{\Delta pc|_z}{q} = \int_{-\infty}^{\infty} E_z e^{-j\kappa z} dz$$

$$\frac{\Delta pc|_y}{q} = \int_{-\infty}^{\infty} \left(E_y - \eta H_x \right) e^{-j\kappa z} dz$$

Travelling Wave Tubes (TWTs)

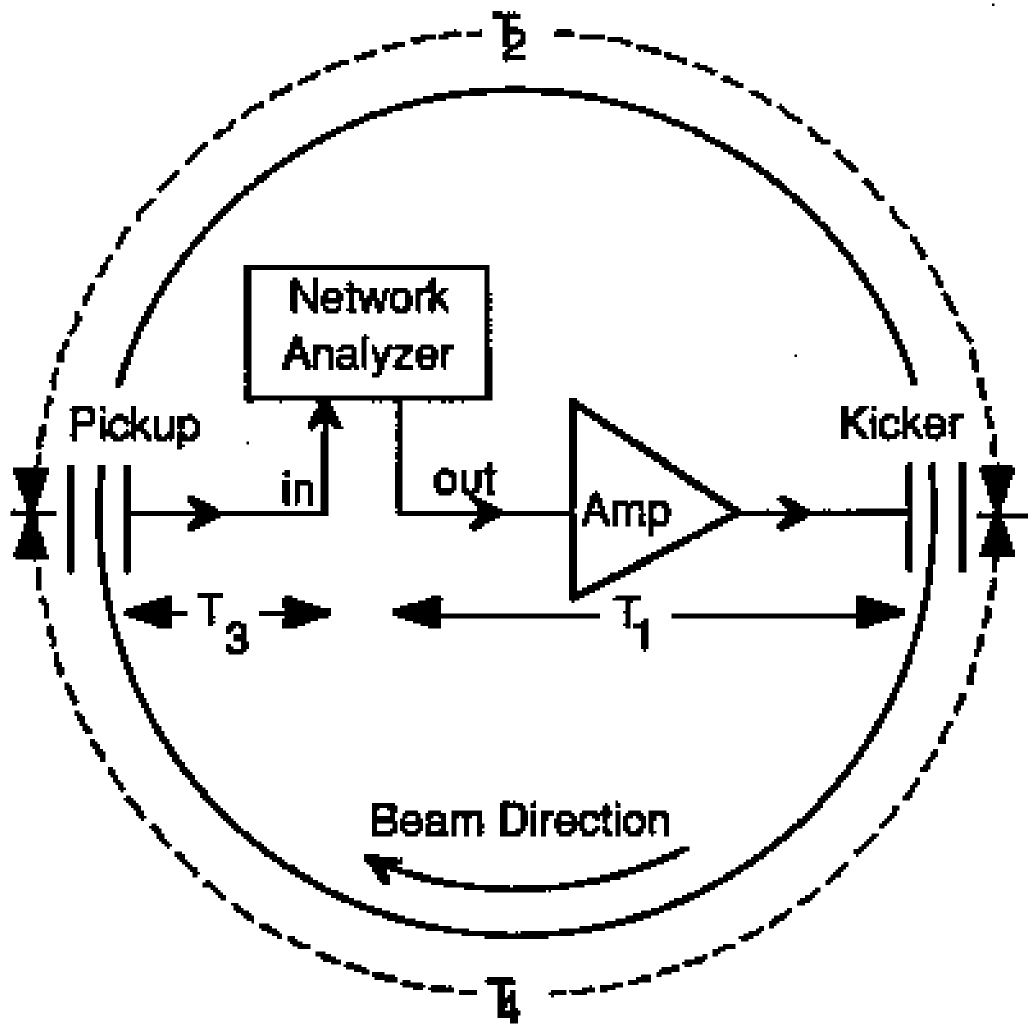
- The power amplifier for cooling must have a large bandwidth
- TWTs can have bandwidths as large as an octave and gain greater than 40 dB
- TWTs have a helix which wraps around an electron beam
  - The helix acts to slow down the electromagnetic wave (RF) that is traveling along the helix from input to output
  - The helix slows down the wave so that the phase velocity of the wave matches the velocity of the electron beam
- At the input, the RF modulates the electron beam velocity
- The modulation in electron beam velocity results in modulation of electron beam density.
- Since the velocities are matched the modulation in electron beam density adds energy to the RF wave

Traveling Wave Tube (TWT)



System Phasing

- For the system to work
  - $T_2 = T_1 + T_3$
- We know that
  - $T_r = T_2 + T_4$
- We measure
  - $T_{meas} = T_1 + T_3 + T_4$
- We adjust T_1 & T_3
  - $T_{meas} = T_r$

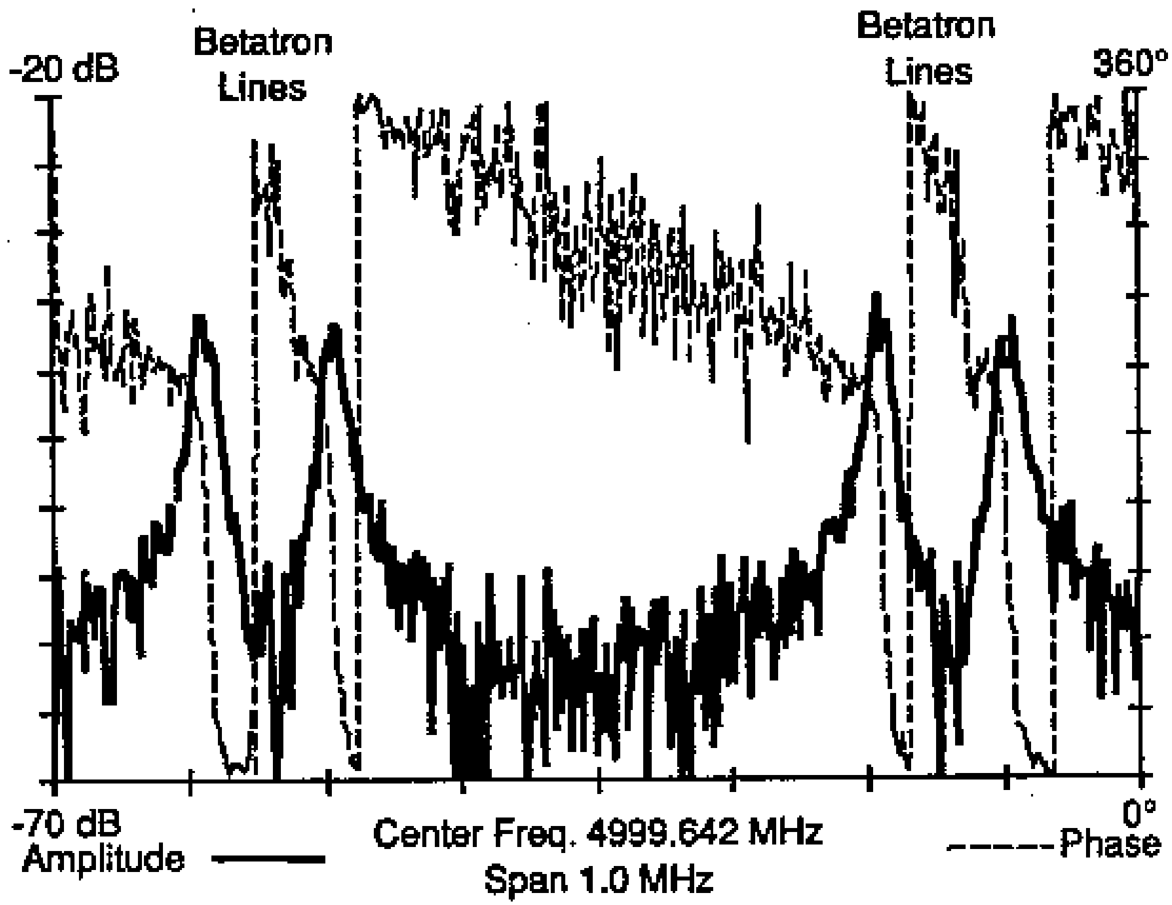


Network Analyzer Measurements

- The output of a network analyzer (NA) sends a single frequency sine wave out to the kicker
- This sine wave modulates the beam
  - Transverse: mini-wiggles the beam
  - Longitudinal: mini-bunches the beam
- The beam responds only if the frequency sent out by the analyzer is close to one of its resonant frequencies
  - Transverse: Betatron sidebands
  - Longitudinal: Revolution lines
- The pickup senses the beam modulation and sends the signal to the input of the NA.
- The NA compares the phase and amplitude of the input sine wave to the output sine wave and displays the vector results

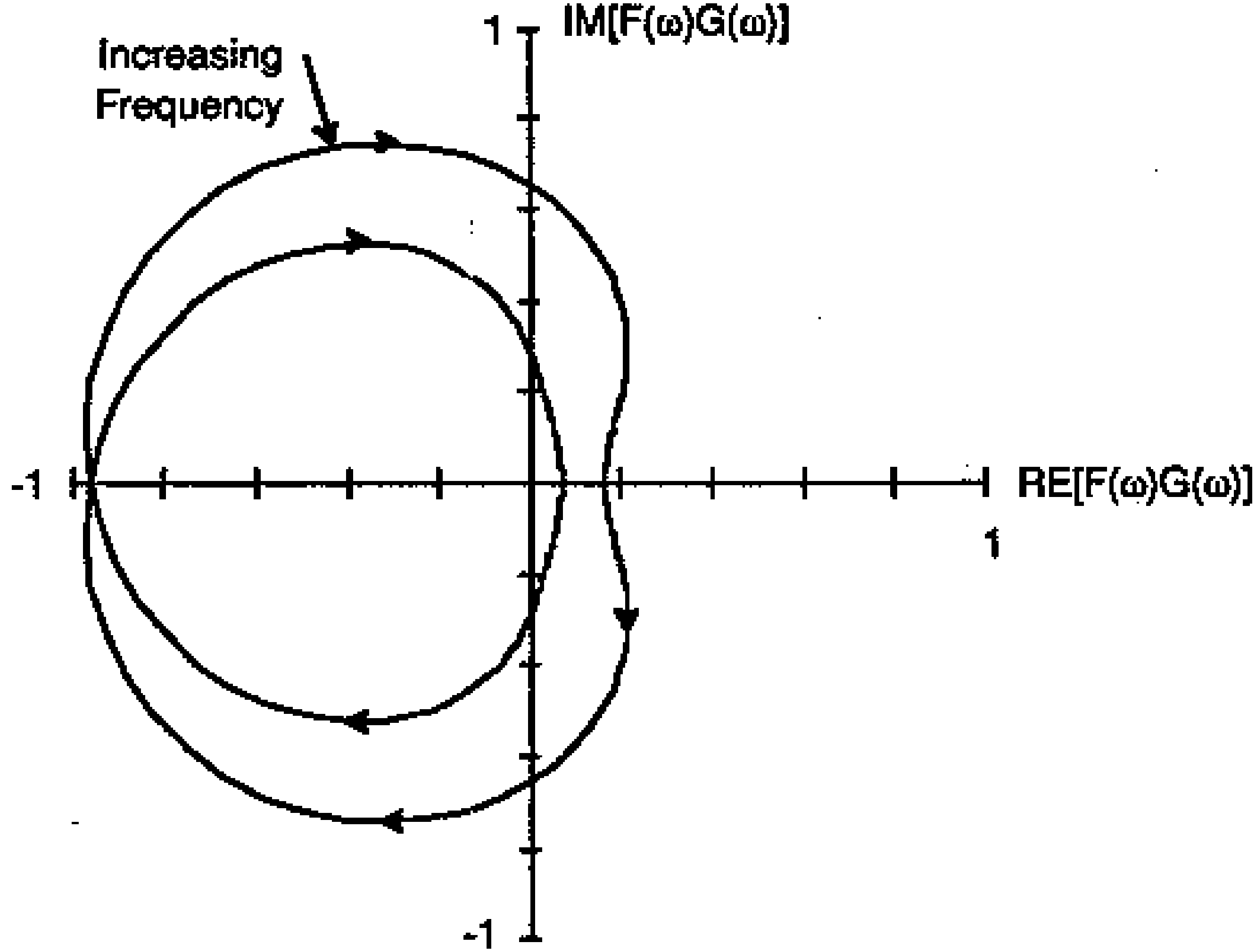
Single Band Transfer Function

- The beam responds only if the frequency sent out by the analyzer is close to one of its resonant frequencies
  - Transverse: Betatron sidebands
  - Longitudinal: Revolution lines

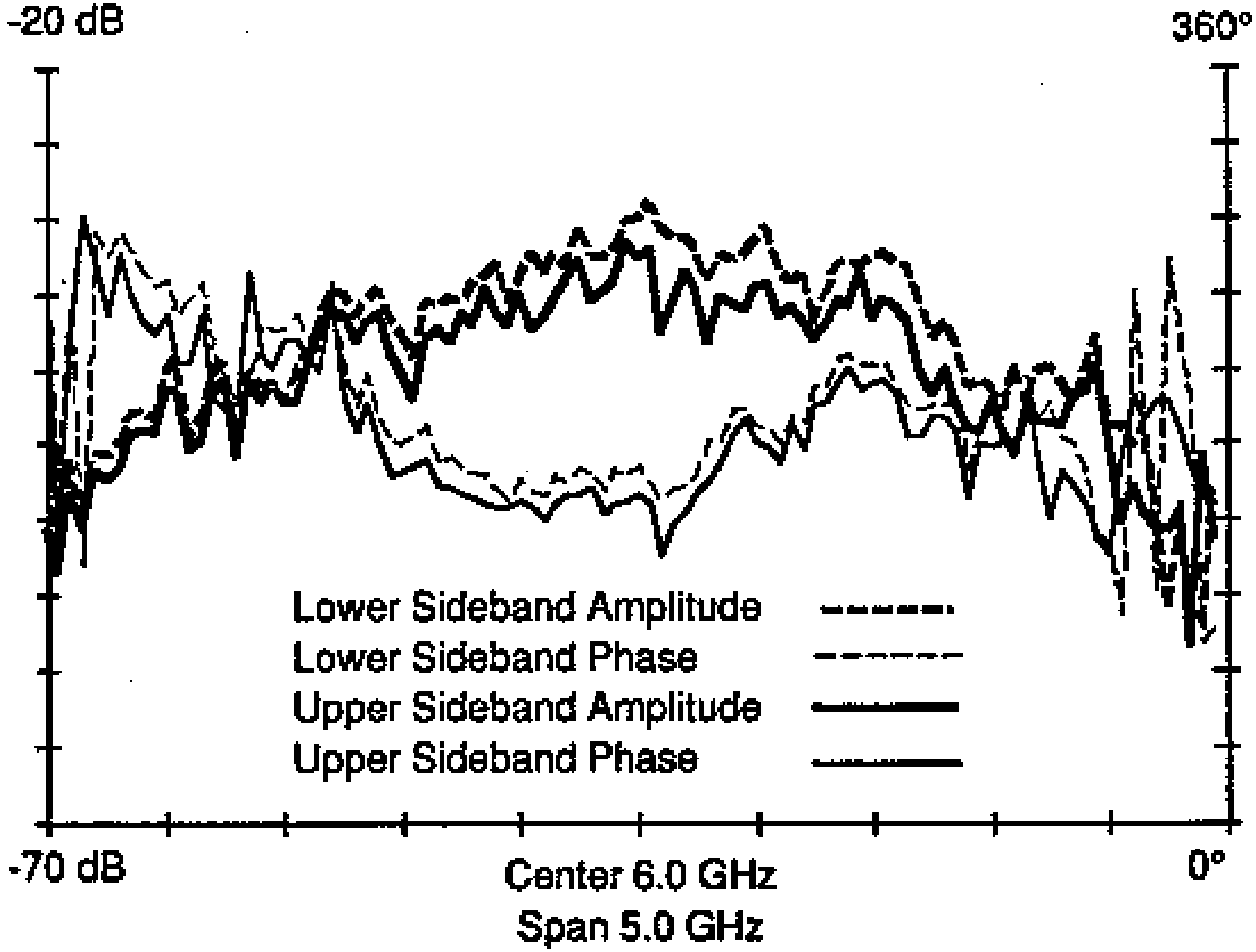


Nyquist Stability Plot

- Another way of displaying the single band transfer function
- If the trajectory encircles +1, the system will be unstable

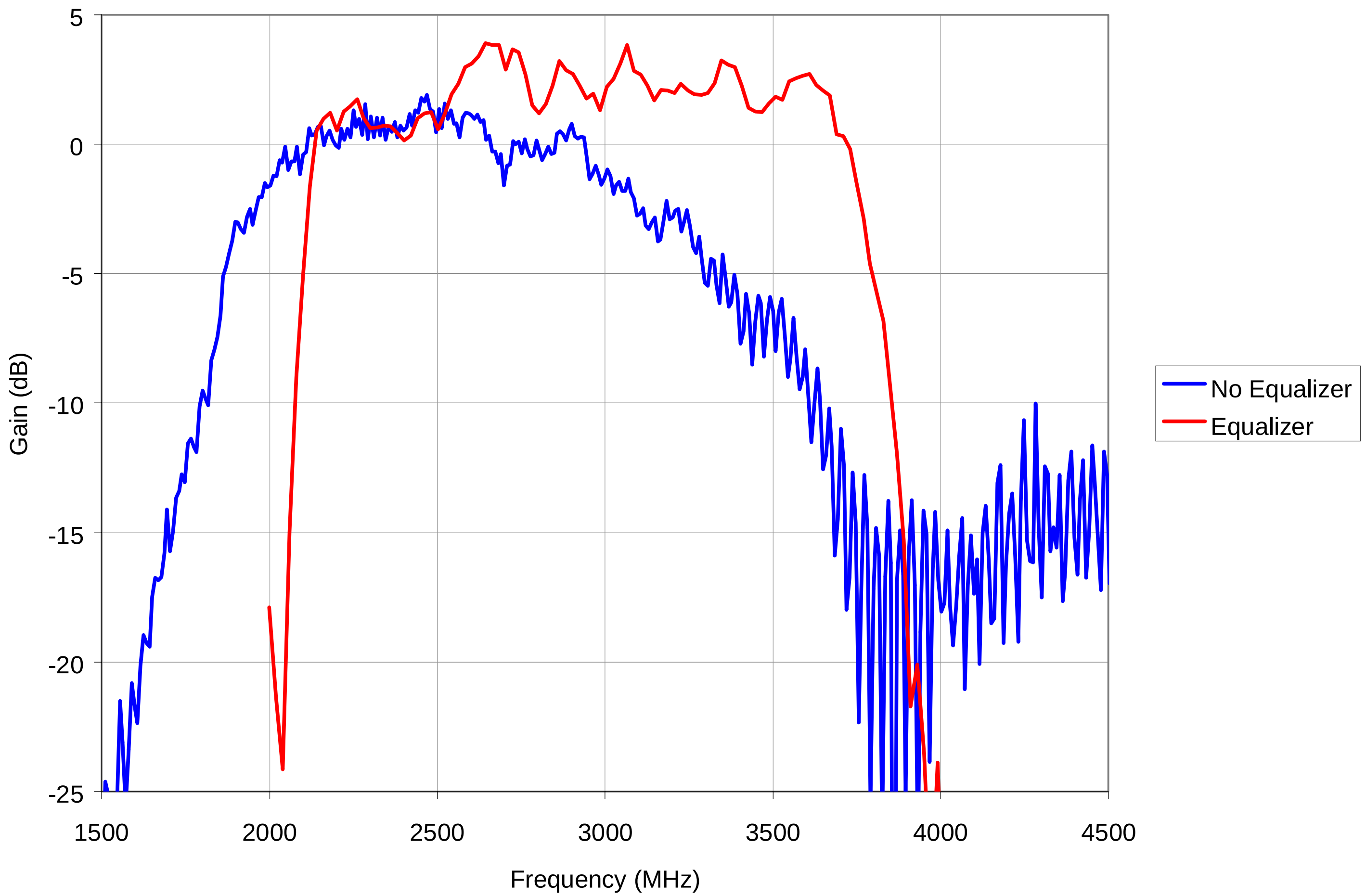


Wideband Transfer Function



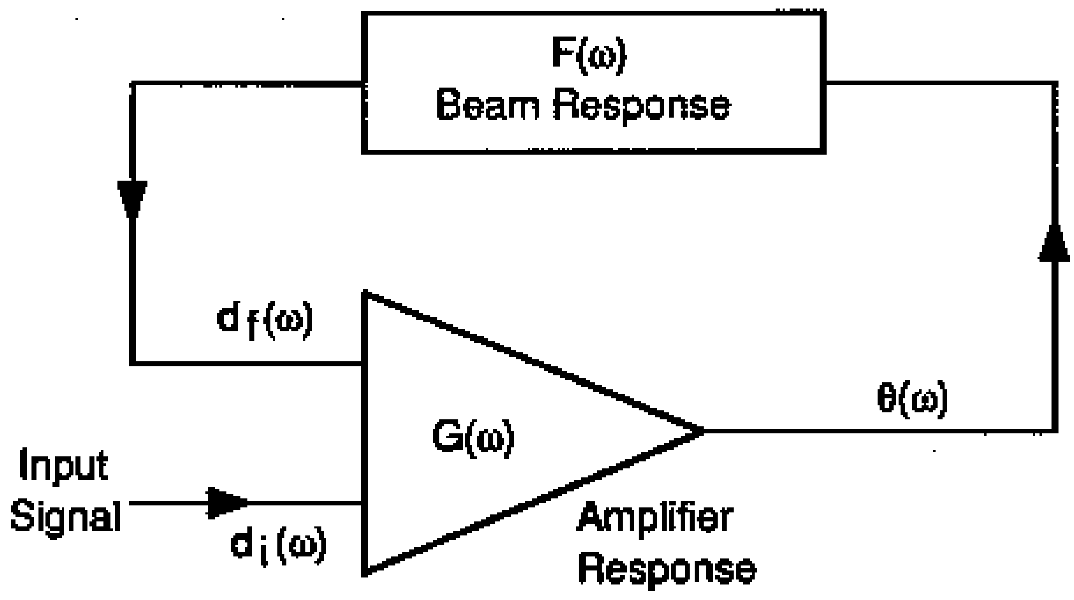
System Equalizers

- The shape of the transfer function can be modified with microwave filters installed in the system trunk.



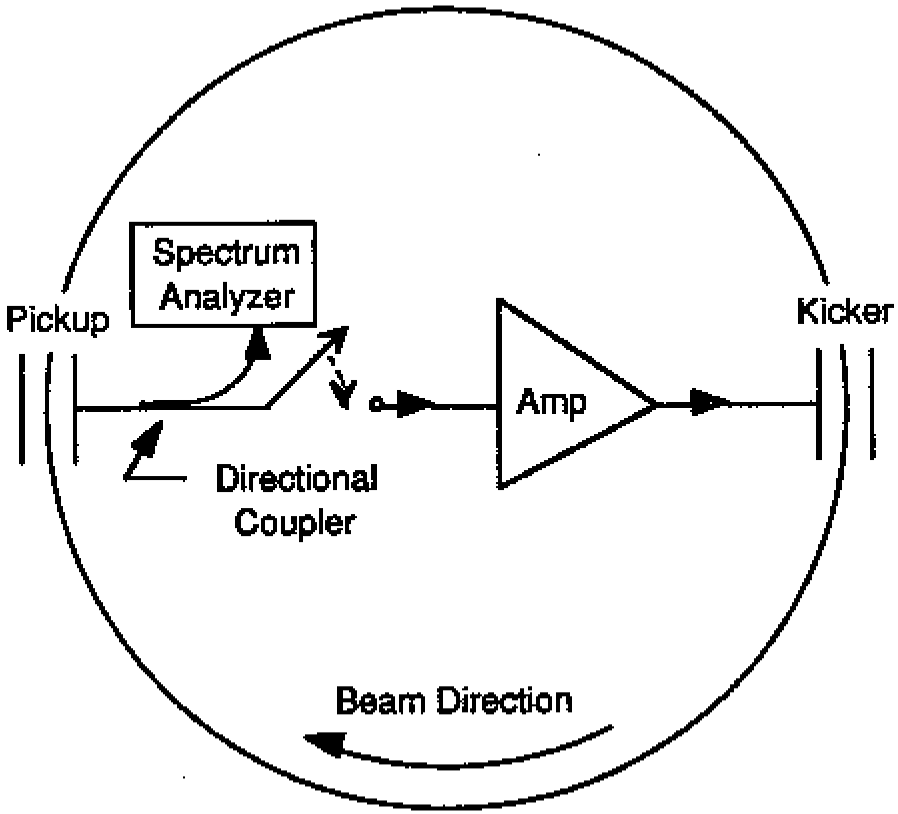
Signal Suppression

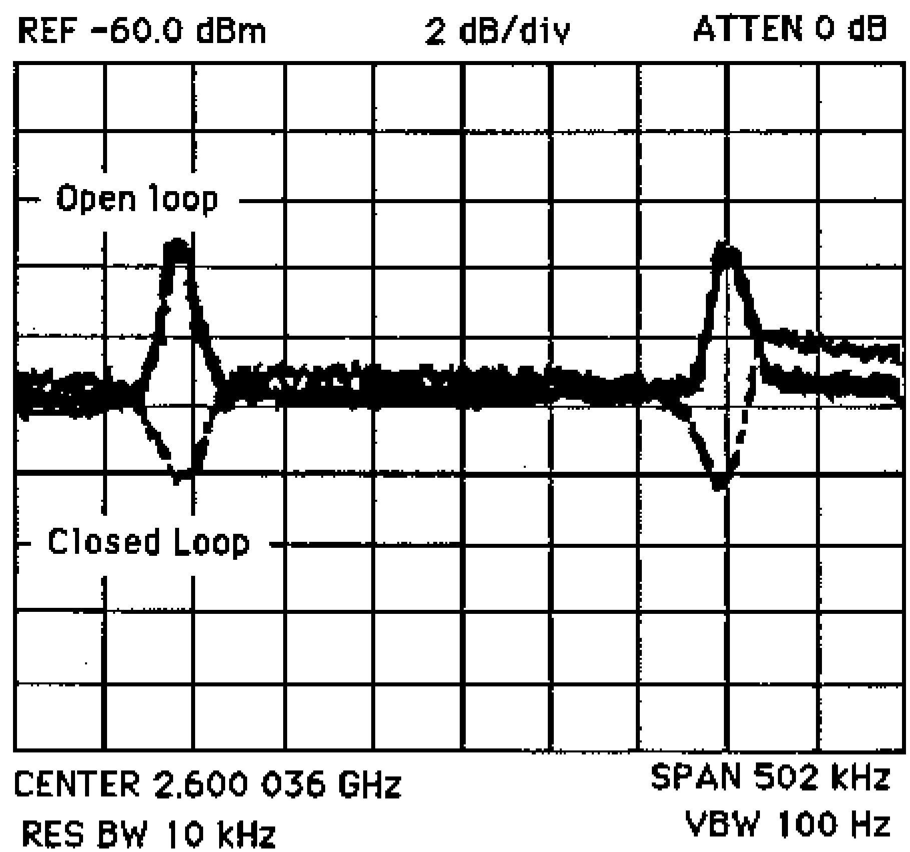
- A stochastic cooling system is a negative feedback system
- With negative feedback, a portion of the output of the system is fed back to the input so as to reduce the signal on the output
- The amount the output is suppressed is proportional to the strength of the feedback



Signal Suppression Measurements

- At optimum gain for a cooling system, the signal is suppressed by 6 dB (a factor of 2)

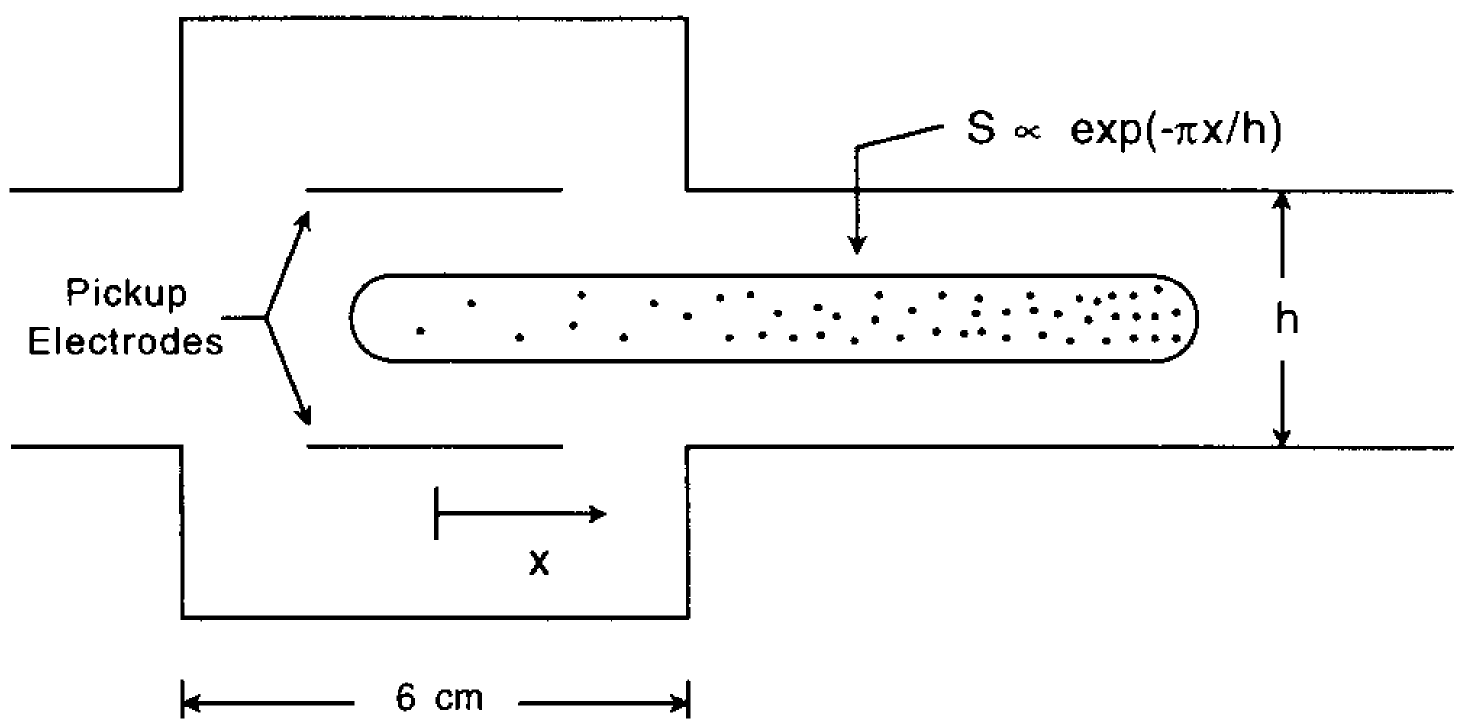




Palmer Momentum Cooling

- Pickups in high dispersion and low beta measure a particle's position which is proportional to the particle's energy.
- Kickers in low (zero) dispersion are wired in sum mode so as to give a longitudinal kick (Electric field in the direction of the particle's motion.
- Advantages
  - Very stable
  - Resistant to bad mixing
- Disadvantages
  - low signal to noise
  - Lattice must have high dispersion

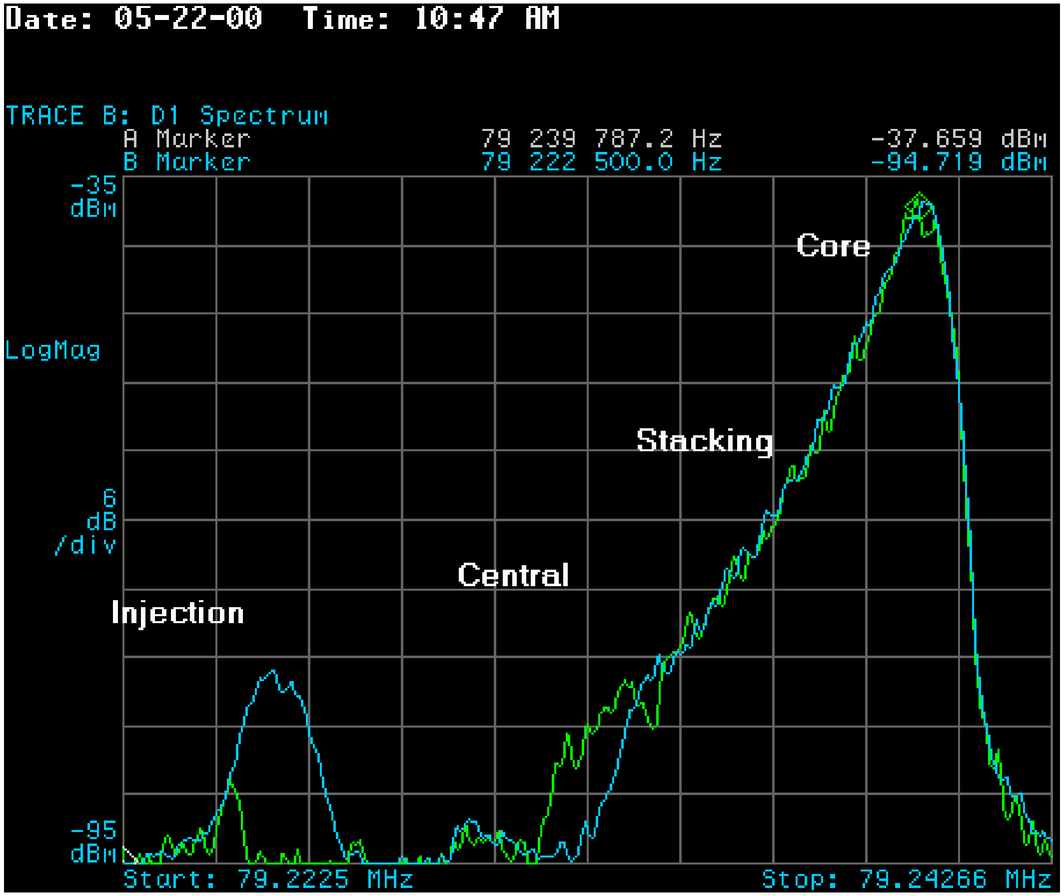
FNAL Accumulator StackTail Pickup



FNAL Accumulator StackTail Profile

$$\Phi_0 = \frac{|\eta|}{4} \frac{W^2}{f_0} \frac{E_d}{pc} \frac{1}{\ln\left(f_{max}/f_{min}\right)}$$

Φ_0 = particle flux
E_d = exponential slope



Filter Momentum Cooling

- Pickups in low dispersion are wired in sum mode.
- Correlator notch filter in trunk acts like an analog 1 turn memory which detect if revolution period of particle is different from desired revolution period
- Kickers in low (zero) dispersion are wired in sum mode so as to give a longitudinal kick
- Advantages
  - Good signal to noise
  - No high dispersion needed in lattice
- Disadvantages
  - Bad mixing because of additional phase slope of filter
  - Closer to instability because of filter phase slope

Correlator Notch Filter

- Delay of long delay line equals desired revolution period
- Delay line made of:
  - Superconducting cable
  - Optical Fiber
  - Bulk Acoustic Wave devices